EVIDENTIAL REASONING IN SUPPORT OF COUNTERINSURGENCY INTELLIGENCE OPERATIONS:

Combining Evidence from Disparate Sources

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Abstract

Perhaps the most difficult problem facing designers of command and control systems is that of combining evidence from disparate sources to form an agreed recognized picture of the environment – especially in support of counterinsurgency operations. More uncertainty exists in counterinsurgency operations and this exacerbates the process. The difficulty generally occurs when we attempt to combine intelligence reports to discern any aspect of the enemy's operations. The combining techniques used in these applications are usually manual assessments of probabilistic estimates. Assuming the probabilities are available, the dynamic nature of counterinsurgency operations generally leads us to the use of Bayesian techniques. The difficulties with Bayesian analysis center on two problems: (1) rapid convergence on a single hypothesis and (2) the need to assess the effect of evidence on the probability of all hypotheses. These and other problems with Baysian analysis have led researchers to search for alternative methods of analysis. One of these is the Shafer-Dempster belief function methodology. Belief functions avoid these difficulties by allowing belief to "grow", i.e., it is acceptable for total belief in all hypotheses and their disjunctions to exceed 1. In this way, rapid convergence is obviated. In addition, there is no need to evaluate conditional probabilities at each iteration. However, belief functions have a few problems of their own that must be addressed before implementation. The most serious is the Dempster rule of combination. This is the belief function equivalent to Bayes rule. With the Dempster rule however, the combinatorial complexity can be prohibitive if the number of hypotheses is large. There are heuristics to mitigate the severity however. Another problem is dealing with disconfirming evidence. Usually one or the other sources must be accepted and the other discarded. In this paper we develop the Shafer-Dempster combining technique and apply it to a simple counterinsurgency example.

INTRODUCTION

Perhaps the most difficult problem facing military intelligence operations in support of counterinsurgency operations is that of combining evidence from disparate sources to form an agreed assessment of insurgent operations. In those cases in which the evidence arrives in the form of electronic signals, standard signal processing techniques, (Oppenheim et al [1]) can be and are used effectively. This is typical of such command and control tasks as tracking and target acquisition (Blackman [2]).

The real problem however, occurs at a much higher level of aggregation. The problem of situational awareness, to include a confident view of insurgent activity and plans, demands that the recognized picture presented to the decision maker be composed of evidence from multiple sources and sensors. The combining techniques used in these applications usually devolve to probabilistic estimates of possible actions the enemy might take. The dynamic nature of counterinsurgency operations leads us to the use of Bayesian techniques as described by Pearl [3] and others.

The difficulties with Bayesian analysis center on two problems: rapid convergence on a single hypothesis and the need to assess the effect of evidence on the probability of all hypotheses. Rapid convergence means that even strong disconfirming evidence arriving late will be ignored. The second problem is at the heart of the Bayesian inversion formula. It requires that we assess the likelihood that the evidence just presented might occur conditioned on all possible hypotheses. Techniques have been suggested to accommodate these difficulties (see e.g., Perry and Stephanou [4]).

These and other problems with Bayesian analysis have led researchers to search for alternative methods of analysis. One of these is the Shafer-Dempster belief function methodology (Pearl [3] and Shafer [6]). This paper summarizes this approach and states the requirements for its implementation in support of intelligence in counterinsurgency operations.

A LESS RESTRICTIVE BAYES

In his book, A Mathematical Theory of Evidence, (Shafer [6]) Glenn Shafer distinguishes between probability and belief:

> First, to have a degree of belief in a proposition is to commit a portion of one's belief to it. And second, whenever one commits only a portion of one's belief to a proposition, one must commit the remainder to its negation. The obvious way to obtain a more flexible and realistic picture is to discard the second of those features while retaining the first. … this leads to the theory of *belief functions.*

This is the essence of a fundamental difficulty associated with using probabilities to express levels of support for a proposition. The axiomatic definition of probability forces us to insist that the probability of an event occurring is the complement of its not occurring. Unfortunately, humans do not always reason in this way unless forced to do so by analysts who confront them with the inconsistency of their thought. For example, suppose your local weatherman tells you that it will rain in your area tomorrow. From his past predictions you subjectively assess him to be 70% accurate. From this you can justify a belief of 70% that it will rain in your area tomorrow but only 0% that it will not rain tomorrow. That is, you have no evidence to support the proposition that it will not rain and therefore, unlike probabilities (where we would assess 30% to the likelihood that it will not rain tomorrow), the 70% and 0%, do not add to 100%. Together then these two constitute a belief function.[1]

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[1] Adapted from Shafer [10]

The theory of *belief functions* is extremely useful in representing support levels for hypotheses in the face of varying degrees of evidence. It is most useful in situational awareness because it allows for support levels for hypotheses to sum to values other than 1.

EMPLACING IMPROVISED EXPLOSIVE DEVICES

We illustrate the use and usefulness of belief functions with a simple example. In Iraq until recently, insurgents turned to a rather simple weapon to harass U.S. military units. Although several versions have been used, the most common improvised explosive device (IED) is the roadside bomb (see Figure 1). These are generally artillery munitions detonated either on command using a remote triggering device, or by some form of trip wire or pressure plate. In counterinsurgency operations in Iraq, considerable time and effort within the intelligence community is devoted to gathering evidence that might point to where these devices are emplaced.

Figure 1: 152 MM Artillery Projectile Hidden in a Burlap Bag

In addition to locating emplaced IEDs, considerable effort is devoted to breaking the IED event-chain. The IED event-chain consists of the sequence of actions the enemy must take from financing the equipment and personnel to construct, transport, emplace and detonate an IED to the detonation itself. Figure 2 illustrates the event chain activities. Each activity contributes to a successful detonation. Breaking the chain requires gathering intelligence (evidence) from sensors and sources operating in the area. In this paper we focus on emplacement only, however, as the diagram suggests, this is only one of several activities where the event chain might be broken.[2] Enemy post-detonation activity generally consists of obtaining and broadcasting videos of the detonation to be used in propaganda activities.

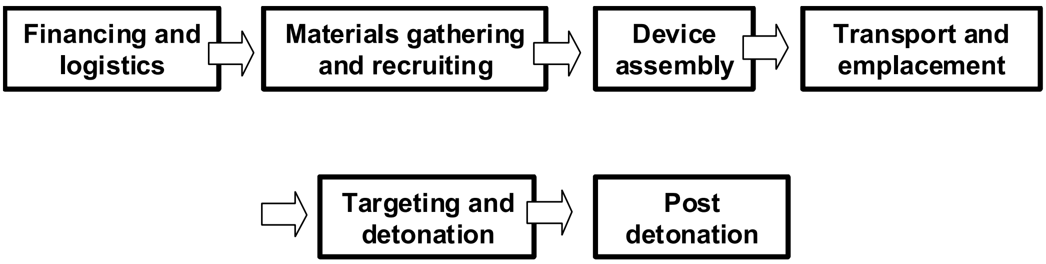

Figure 2: The IED Event Chain

In our hypothetical problem, the local friendly commander is concerned about a particularly busy intersection where several convoys must pass daily. Historically, this has been a favorite location for emplaced IEDs. One solution might be for the local friendly commander to deploy a small force to continuously guard the intersection. However, this is not practical because it would interfere with his mission. That is, his soldiers are needed for other operations. In addition, continuously monitoring the intersection would likely drive the insurgents to emplace the IED at another site. Nevertheless he would still like to know in advance if an IED is emplaced at the intersection before the day's operations begin.

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2 For a fuller discussion of counterinsurgency operations and the event chain see Perry and Gordon [5].

We assume that the time available to emplace an IED is such that the enemy can emplace only one per day. The time available to emplace an IED is a function of friendly traffic on the route so most are emplaced during the hours of darkness. In essence, then, the friendly commander is confronted with three possible enemy courses of action which we will state as hypotheses:

h_1 = The enemy emplaces an IED at intersection.

h_2 = The enemy emplaces an IED elsewhere.

h_3 = The enemy does not emplace an IED.

For our purposes, this set is therefore collectively exhaustive and mutually exclusive. The set of these courses of action constitutes the hypothesis set: $\mathbf{H} = \{h_1, h_2, h_3\}$.

We would expect the intelligence community to routinely gather evidence from multiple sources to support one or more of these hypotheses. Note that unlike in Bayesian analysis, we use the term "one or more." That is, we are not restricted to assessing belief only in the hypotheses themselves, but rather in all logical disjunctions of these hypotheses. We explore this feature of belief functions more fully later.

DISCERNING TRUTH

The set of hypotheses, $\mathbf{H} = \{h_1, h_2, h_3\}$, is referred to as the *frame of discernment*. If a logical proposition consists of a subset of **H**, then we say the *frame discerns* the proposition. For example, a proposition such as "the enemy will emplace an IED" is the logical disjunction of the two propositions: h_1, "the enemy emplaces an IED at the intersection" and h_2, "the enemy will emplace an IED elsewhere" The set theoretic representation of the disjunction is $\{h_1, h_2\}$, a subset of **H.** Note that the disjunctions $\{h_1, h_3\}$ and $\{h_2, h_3\}$ are also possible. The former focuses on the belief that an IED will be emplaced at the intersection or that there will be no emplacement. The latter shifts the focus away from the intersection to somewhere else in the area of operations. Assessing belief in either expresses uncertainty whether an IED has been emplaced or not.

In general, we exploit the correspondence between propositions and subsets so that the logical notions of conjunction, disjunction, implication and negation map into set-theoretic notions of intersection, union, inclusion and complementation.

This leads us to examine all of the possible subsets of the frame of discernment: $2^{\mathbf{H}} = \{\phi, \mathbf{H}, \{h_1\}, \{h_2\}, \{h_3\}, \{h_1, h_2\}, \{h_1, h_3\}, \{h_2, h_3\}\}$. At the logical level, this set represents all of the propositions A that imply the frame **H**. That is, **H** "includes" A. The set $2^{\mathbf{H}}$ is referred to as the power set because its cardinality is $2^{|\mathbf{H}|}$. Each of these propositions can be supported at some level based on evidence obtained from sensors and sources reports. Next we formally define the level of support, $m(A)$, for the proposition A.

Definition: If **H** is a frame of discernment, then a function, $m : 2^{\mathbf{H}} \to [0,1]$ is called a *basic probability assignment* number when:

- $m(\phi) = 0$ and
- $\sum_{A \in \mathbf{H}} m(A) = 1$.

In this formulation, $m(A)$ represents the belief committed to A only. No information concerning the support levels for the subsets of A is available from this assignment. For example, if *A* represents the proposition "the enemy will emplace an IED", then regardless of the support level for *A*, we draw no conclusions about whether the IED will be emplaced at the intersection or elsewhere. Suppose the support level for *A* is .3. Then $m(A) = m(\{h_1, h_2\}) = .3$.

Note the difference between this formulation and the probability approach. In general, if $A = \{h_1, h_2\}$ then $P(A) = P(h_1) + P(h_2) - P(h_1 \cap h_2)$. Since h_1 and h_2 are mutually exclusive in this example, we must impose the identity $P(A) = P(h_1) + P(h_2) = .3$. This restriction does not apply with belief functions.

Total Belief

However, it is desirable to examine the *total belief* committed to the proposition *A*. That is, the belief committed to *A* and all logical propositions that imply *A*. We use the notation $Bel(A)$ to represent the total belief assigned to *A*, calculated as the sum of basic probability assignments for all of the subsets of *A* or more formally, $Bel(A) = \sum_{B \subset A} m(B)$. Now we have that $Bel(\{h_1, h_2\}) = m(\{h_1\}) + m(\{h_2\}) + m(\{h_1, h_2\})$. Note that this construct admits belief levels that do not necessarily sum to 1.

Vacuous Belief

Another interesting contrast between probability representations and Belief Functions is in the comparative representation of total uncertainty. If there is no evidence to support any subset (proposition), $A \subset \mathbf{H}$, then the frame absorbs all the belief and $m(\mathbf{H}) = 1$ and $m(A) = 0$ for all $A \neq \mathbf{H}$. This means that the total belief for all propositions, except that "the enemy will emplace an IED or the enemy will not emplace an IED" is 0 or: $Bel(\mathbf{H}) = 1$ and $Bel(A) = 0$ for all $A \neq \mathbf{H}$.

Allowing the frame to absorb all of the belief under conditions of total uncertainty is equivalent to applying a uniform probability assignment on the hypothesis sample space in Bayesian analysis or $P(h_i) = 1/n$ where n is the number of hypotheses. The difference between the two is significant in that the former allows us to reflect our ignorance by assigning support levels of 0 to all propositions in the frame whereas Bayesian analysis forces us to assign probability (and therefore belief) of $1/n$ to every hypothesis in the sample space.

BELIEF FUNCTIONS AND THEIR APPLICATION

$Bel : 2^{\mathbf{H}} \rightarrow [0,1]$ is a *belief function* over **H** if it satisfies the following conditions:

1. $Bel(\mathbf{H}) = 1$,
2. $Bel(\phi) = 0$, and
3. For every $n > 0$ and every collection $A_1, A_2, \ldots A_n$ subsets of **H**,

$$Bel(A_1 \cup \ldots \cup A_n) \geq \sum_i Bel(A_i) - \sum_{i<j} Bel(A_i \cap A_j) + - \cdots + (-1)^{n+1} Bel(A_1 \cap \cdots \cap A_n)$$

The first two properties are consistent with the axiomatic definition of probability. The third is where the two depart. If the frame has cardinality k for example, and $A_1 = \{h_1, h_5, h_{10}\}$ and $A_2 = \{h_3, h_5\}$ then the total belief committed to the disjunction of the two satisfies the following inequality: $Bel(A_1 \cup A_2) = Bel(A_1) + Bel(A_2) - Bel(A_1 \cap A_2) = Bel(\{h_1, h_5, h_{10}\}) + Bel(\{h_3, h_5\}) - Bel\{h_5\}$.

Note that the belief function is Bayesian if we have that $Bel(A_1 \cup A_2) = Bel(A_1) + Bel(A_2)$ for all $A_1, A_2 \in \mathbf{H}$ whenever $A_1 \cap A_2 = \phi$.

To make this a bit more concrete, suppose the evidence from a single source results in the support levels for the following propositions (hypotheses):

- "The enemy will emplace an IED at the intersection" $m(\{h_1\}) = .2$
- "The enemy will emplace an IED somewhere else" $m(\{h_2\}) = 0$
- "The enemy will emplace an IED" $m(\{h_1, h_2\}) = .3$
- "The enemy will not emplace an IED" $m(\{h_3\}) = .2$
- All other propositions $m(\{h_1, h_3\}) = m(\{h_2, h_3\}) = 0$ and $m(\mathbf{H}) = .3$

This example illustrates the methodology. For now, we assume that the evidence we have gathered comes from a single source. We deal with evidence from multiple sources later in the discussion of Dempster's rule of combination. The evidence received allows us to make basic probability assignments to some of the subsets of the frame.

Note that it is possible to support disjunctions independently. That is, if the assignments were probabilities, then we would have that $m(\{h_1, h_2\}) = m(\{h_1\}) + m(\{h_2\}) = .2$, and not 0.3. The restriction that the assignments sum to 1.0 forces us to assign 0.3 to the frame. Next, we observe that the belief functions arising from these assignments produce some interesting results.

- $Bel(\{h_1\}) = m(\{h_1\}) = .2$,
- $Bel(\{h_2\}) = m(\{h_2\}) = 0$,
- $Bel(\{h_3\}) = m(\{h_3\}) = .2$,
- $Bel(\{h_1, h_2\}) = m(\{h_1, h_2\}) + m(\{h_1\}) + m(\{h_2\}) = .5$,

- $Bel(\{h_1, h_3\}) = m(\{h_1, h_3\}) + m(\{h_1\}) + m(\{h_3\}) = .4$, and
- $Bel(\{h_2, h_3\}) = m(\{h_2, h_3\}) + m(\{h_2\}) + m(\{h_3\}) = .2$

Note that the sum of the total beliefs of all subsets of the frame is considerably greater than 1. We can also apply the third condition for belief functions to this example. We have for example that: $Bel(\{h_1, h_2\} \cup \{h_1, h_3\} \cup \{h_2, h_3\}) = Bel(\mathbf{H}) = 1$, and that $Bel(\{h_1\} \cup \{h_1, h_2\}) = Bel(\{h_1, h_2\}) = .5$. Evaluating the right side of the inequality we get:

$$\begin{aligned} Bel(\{h_1\} \cup \{h_1, h_2\}) &= Bel(\{h_1\}) + Bel(\{h_1, h_2\}) - Bel(\{h_1\} \cap \{h_1, h_2\}) \\ &= Bel(\{h_1\}) + Bel(\{h_1, h_2\}) - Bel(\{h_1\}) = .5, \end{aligned}$$

and therefore the equality condition holds. A more interesting case is to evaluate the right side of the inequality for $Bel(\{h_1, h_2\} \cup \{h_1, h_3\} \cup \{h_2, h_3\}) = Bel(\mathbf{H}) = 1$. This gives us:

$$\begin{aligned} Bel(\{h_1, h_2\} \cup \{h_1, h_3\} \cup \{h_2, h_3\}) &= Bel(\{h_1, h_2\}) + Bel(\{h_1, h_3\}) + Bel(\{h_2, h_3\}) \\ &\quad - Bel(\{h_1, h_2\} \cap \{h_1, h_3\}) - Bel(\{h_1, h_2\} \cap \{h_2, h_3\}) \\ &\quad - Bel(\{h_1, h_3\} \cap \{h_2, h_3\}) + Bel(\{h_1, h_2\} \cap \{h_1, h_3\} \cap \{h_2, h_3\}) \\ &= Bel(\{h_1, h_2\}) + Bel(\{h_1, h_3\}) + Bel(\{h_2, h_3\}) \\ &\quad - Bel(\{h_1\}) - Bel(\{h_2\}) - Bel(\{h_3\}) + Bel(\phi) \\ &= .5 + .4 + .2 - .2 - 0 - .2 + 0 = .7 \end{aligned}$$

Therefore, in this case the inequality holds.

FOCAL ELEMENTS

Belief is concentrated in focal elements. A *focal element* is a subset of **H** that has some (non-zero) support, that is, if $A \subset \mathbf{H}$, then A is a focal element if and only if $m(A) > 0$. In the IED example, all but two of the subsets of **H** are focal elements. There is no support for the logically inconsistent propositions $\{h_1, h_3\}$ and $\{h_2, h_3\}$, that is, it is illogical to assign belief to the proposition that an IED can be emplaced and not emplaced at the same time. The union of the focal elements is referred to as the *core*. In the IED example, the core is the frame. This is important later when we discuss the rules of combining evidence fro two or more sources.

The combining algorithm only deals with focal elements. Therefore it is always easier to combine evidence from disparate sources if the number of focal elements is small. This requires that belief be concentrated in as few subsets of the frame as possible.

We can think of the focal element in terms of the basic probability assignment. Figure 3 is a geometric depiction of the probability assigned to a subset. We think of the probability mass (0.3) as being confined to the set $\{h_1, h_2\}$ and capable of moving freely among the elements of the set. That is, belief is "focused" on the set $\{h_1, h_2\}$ and not on the elements of the set.

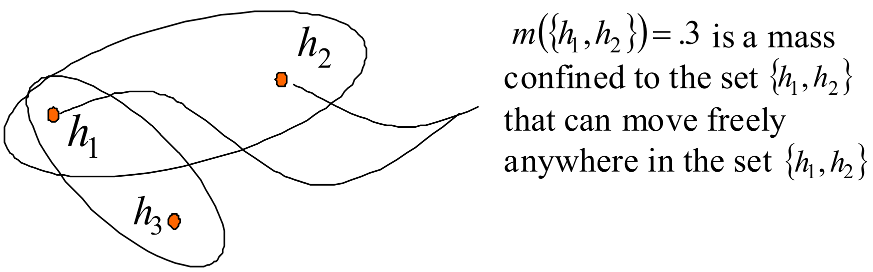

Figure 3: Focal Elements

DEMPSTER'S RULE

So far we have examined a methodology for assessing the belief we are willing to assign to all subsets of a frame of discernment. The levels of belief are derived from the evidence provided by a single observation of a single source. The next step is to examine a method for combining the evidence from multiple sources, both similar and disparate or from repeated observations from a single source. In Bayesian analysis, we conditionally update the probabilities on the hypotheses based on the collected evidence. For belief functions, we apply Dempster's rule of combination (Dempster [7]).

Dempster's rule allows us to focus on the focal elements of the belief functions developed from the evidence produced from two sources and compute an *orthogonal sum* of the two that results in a third. The combined belief function can then be combined with yet another belief function, and so forth.

We illustrate the process with the IED example developed so far. We assume we have two belief functions defined on the same frame of discernment, $\mathbf{H} = \{h_1, h_2, h_3\}$. The evidence could have come from disparate or similar sources.[3] We also assume that the two assessments are independent. Regardless, we denote the first Bel_1 and the second Bel_2 and depict the orthogonal sum as $Bel_1 \oplus Bel_2$.

The basic probability support levels for the focal elements from the two sources are as follows. Note that we are taking Bel_1 to be the belief function already developed. Also note that only propositions with support are listed as focal elements, consistent with its definition.

- $Bel_1 : m_1(\{h_1\}) = .2, m_1(\{h_3\}) = .2, m_1(\{h_1, h_2\}) = .3, m_1(\mathbf{H}) = .3$
- $Bel_2 : m_2(\{h_3\}) = .2, m_2(\{h_1, h_2\}) = .3, m_2(\mathbf{H}) = .5$

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[3] For example, human intelligence reports could be used to generate the first belief function and unmanned aerial vehicle observations for the second or the second may be derived from a second, subsequent human intelligence report.

The combined support level for the focal element *A*, $m_{1,2}(A)$, for any two focal elements is the normalized sum of the product of focal elements from both belief functions whose intersection is *A*. The normalizing divisor is the complement of the product of the support levels for all disjoint focal elements. Mathematically we get:

$$(1)\ m_{1,2}(A) = \frac{\sum\limits_{\substack{i,j \\ A_i \cap B_j = A}} m_1(A_i) m_2(B_j)}{1 - \sum\limits_{\substack{i,j \\ A_i \cap B_j = \phi}} m_1(A_i) m_2(B_j)} \text{ provided } \sum_{\substack{i,j \\ A_i \cap B_j = \phi}} m_1(A_i) m_2(B_j) < 1$$

Rather than apply this definition directly, we resort to a simple algorithmic process. The matrix depicted in Table 1 below represents the products of all combinations of support levels between Bel_1 and Bel_2 (the numerator in (1) above). Note that the matrix is not necessarily square (although it is likely to be so if the sensors or sources are similar). We also include the frame because of the requirement that the basic probability assignments sum to 1. Each entry in the table is the product of the support levels for the row and column entries. For example, $m_{1,2}(\{h_3\}) = m_1(\{h_3\}) m_2(\{h_3\}) = .2 \times .2 = .04$. The entries in "()" represent the product of two support levels for disjoint focal elements.

Table 1
Support Level Products

Bel_2 \ Bel_1	$m_1(\{h_1\})$	$m_1(\{h_3\})$	$m_1(\{h_1, h_2\})$	$m_1(\{\mathbf{H}\})$
$m_2(\{h_3\})$	**(.04)**	**.04**	**(.06)**	**.06**
$m_2(\{h_1, h_2\})$	**.06**	**(.06)**	**.09**	**.09**
$m_2(\{\mathbf{H}\})$	**.10**	**.10**	**.15**	**.15**

Next we calculate the normalizing divisor by summing the "disjoint" entries and subtracting from 1. This is the denominator in (1) above.

$$1-\sum_{\substack{i,j \\ A_i \cap B_j=\phi}} m_1(A_i) m_2(B_j) = 1-(.04+.06+.06) = .84$$

Table 2 then represents the normalized entries with support for disjoint focal elements set to 0. This table is used to calculate the combined belief function, $Bel_1 \oplus Bel_2$.

Table 2
Normalized Support Levels

Bel_2 \ Bel_1	$m_1(\{h_1\})$	$m_1(\{h_3\})$	$m_1(\{h_1, h_2\})$	$m_1(\{\mathbf{H}\})$
$m_2(\{h_3\})$	**0**	**.0476**	**0**	**.0714**
$m_2(\{h_1, h_2\})$	**.0714**	**0**	**.1071**	**.1071**
$m_2(\{\mathbf{H}\})$	**.1190**	**.1190**	**.1785**	**.1785**

The combined basic probability assignments for each focal element are calculated as the sum of the entries in Table 2 for which the intersection of the row focal elements and column focal elements are the focal element being evaluated. For example, for the focal element $A = \{h_3\}$ we get the following (from Table 2):

$m_{1,2}(\{h_3\}) = .0476 + .0714 + .1190 = .2380$. In general, the combined probability assignments are calculated from Table 2 as:

$$m_{Bel_1 \oplus Bel_2}(A) = \frac{\sum\limits_{\substack{i,j \\ A_i \cap B_j = A}} m_1(A_i) m_2(B_j)}{1 - \sum\limits_{\substack{i,j \\ A_i \cap B_j = \phi}} m_1(A_i) m_2(B_j)}.$$

The resultant combined support levels for each focal element then is:

$$Bel_1 \oplus Bel_2 : m_{1,2}(\{h_1\}) = .1904, m_{1,2}(\{h_3\}) = .2380,$$
$$m_{1,2}(\{h_1, h_2\}) = .3927, m_{1,2}(\mathbf{H}) = .1785$$

It is interesting to compare the combined support levels to the constituent levels before combining. They are repeated here for convenience:

- $Bel_1 : m_1(\{h_1\}) = .2, m_1(\{h_3\}) = .2, m_1(\{h_1, h_2\}) = .3, m_1(\mathbf{H}) = .3$
- $Bel_2 : m_2(\{h_3\}) = .2, m_2(\{h_1, h_2\}) = .3, m_2(\mathbf{H}) = .5$

For example the focal element $A = \{h_3\}$ has the following constituent support levels: $m_1(\{h_3\}) = m_2(\{h_3\}) = .2$. Compare this to the combined support level: $m_{1,2}(\{h_3\}) = .2380$. Although each belief function supported the proposition at the same level, the result was an actual <u>increase</u> in its combined support. In addition, note that the support level for the frame has decreased considerably. The combination of two independent reports has reduced the level of uncertainty as more belief was assigned to focal elements. We observe the same phenomenon for the proposition $\{h_1, h_2\}$. In contrast, the support for $\{h_1\}$ decreased due to lack of direct Bel_2 support. In fact, the only reason we have support at all is because of the support derived from the support for the proposition $\{h_1, h_2\}$.

Finally, we examine the total belief attributed to each of the propositions in the frame of discernment. In Table 3, the first two columns are the total beliefs from the two independent sources and the third is the total belief produced by the orthogonal sum of the first two. The observations made about the basic probability assignments carry through to the belief functions.

The change in belief for the individual hypotheses, h_1, h_2, h_3, from just these two sources, imply that considerable uncertainty still exists and that more evidence is needed. At the same time, we are able to make statements about the disjunctions in that most of the belief is concentrated there. In other words, our best information is at a higher level of precision than we would like. That is, we have fairly good support (.5831) that an IED will be emplaced, but weak support as to where. In addition, we have support for the propositions that an IED will be emplaced and that it will not be emplaced. This logical inconsistency reflects continuing uncertainty requiring additional intelligence reports. All of this suggests that the belief function process for combining evidence produces assessments at *variable levels of precision* (see Perry and Stephanou [4]).

Table 3
Total Belief

(A)	$Bel_1(A)$	$Bel_2(A)$	$Bel_{1\oplus 2}(A)$
$\{h_1\}$	.2	0	.1904
$\{h_2\}$	0	0	0
$\{h_3\}$	.2	.2	.2380
$\{h_1, h_2\}$	.7	.5	.5831
$\{h_1, h_3\}$	.4	.2	.4284
$\{h_2, h_3\}$	.2	.2	.2380
H	1	1	1

DISAGREEMENT (CONFLICT)

One of the difficulties associated with belief functions is the resolution of conflict. In the IED example, if the conjunction of two propositions has no support, i.e., if the propositions A and B are such that $A \cap B = \phi$, then the fact that either or both are supported in the constituent belief functions gives rise to a conflict. For example, the proposition that the enemy will emplace an IED is $A = \{h_1, h_2\}$. Note from Table 3 that the combined belief function shows considerable support for the proposition. However, at the same time, the proposition that the enemy will not emplace an IED, $B = \{h_3\}$ also has support. Clearly $A \cap B = \phi$. In the combining of the two belief functions, Bel_1 and Bel_2 depicted in Tables 1 and 2, there are three instances where $A \cap B = \phi$. In Table 2, the support levels for all three are 0.

The extent of the conflict is measured by the size of the normalizing divisor, $\kappa = \sum_{\substack{i,j \\ A_i \cap B_j = \phi}} m_1(A_i) m_2(B_j)$. For the example depicted in Table 1, we get $\kappa = .16$. The degree of conflict between the belief functions Bel_1 and Bel_2 is then measured as:

$$Con(Bel_1, Bel_2) = \log\left(\frac{1}{1-\kappa}\right) = -\log(1-\kappa) = -\log(.84) \cong .17.$$

Total conflict occurs when none of the focal elements supported by each of the belief functions have common elements ($\kappa = 1$ and $Con(Bel_1, Bel_2) \to \infty$). This is equivalent to asserting that at least one of the sources reporting is completely "wrong." When this occurs, one of the belief functions must be "believed" and the other discarded. If this occurs after several combinations, then the last evidence to be combined that produces this condition can easily be discarded. In any event, it does cause a problem.

CAUTIONS

There are two shortcomings in using Dempster's rule of combination, one procedural and one methodological. We have alluded to the second already. There is simply no good way to deal with total conflict. Even the existence of partial conflict means that one or more reports are disregarded. Nevertheless, the Dempster rule of combination at least allows us to measure the degree of conflict that exists between reports. In Bayesian updating, the erroneous probability is simply combined with the apriori probability using Bayes rule.

The first shortcoming is more problematic but for a very practical reason. The combinatorial complexity of the rule of combination is $O\left(2^{2|\mathbf{H}|}k\right)$, where $|\mathbf{H}|$ is the cardinality of the frame and k is the number of sensor reports to combine.[4] For small frames, i.e., when the number of insurgent emplacement possibilities is small, complexity is no problem. However, as the number increases, the problem magnifies considerably. for $|\mathbf{H}| = 4$ for example, the combinatorial complexity is order 256 for each combination. However, for 5, it increases to 1024 and for 10, it reaches 1,048,576! For large frames, other methods are required. One such method is described in Perry and Stephanou [4]. See also Fixsen [8].

This second requirement is not unique to belief functions. In Bayesian analysis, the probability that a phenomenon might be observed conditioned on the truth of each hypothesis must be assessed at each iteration. In belief function analysis, what is needed is a mapping between all the output from the sensor/source and $A \subset \mathbf{H}$ for all $m(A)$. This is generally a much smaller problem.

Finally, we return the problem of dealing with conflict. Here, there is no good way to proceed except to resort to another method. One such method is Variable Precision Reasoning described in Perry and Stephanou [4]

CONCLUSION

The intelligence problem is one of developing evidence from disparate or similar sources and combining that evidence to generate a picture of enemy activity. In using either belief functions or Bayesian methods to generate the assessments, it is necessary to reduce the evidence collected to subjective probabilities.

Using Bayesian methods, belief is assessed at the hypothesis level only. That is, if the set of hypotheses is $\mathbf{H} = \{h_1, h_2, h_3\}$, then we must assign probabilities, $P(h_1), P(h_2),$ and $P(h_3)$. Belief in disjunctives is calculated, not assessed as in belief functions. That is $P(h_1 \cup h_2) = P(h_1) + P(h_2) - P(h_1 \cap h_2)$. In addition, since probabilities must sum to 1, ignorance results in the assignment of belief of $1/n$ for each hypothesis. With belief functions, belief can be assessed at any level of precision, that is, for any subset of **H**. In addition, vacuous belief assigns all the belief to the frame, **H** and no belief to propositions contained in **H**.

Combining the evidence collected is essentially the process of updating the current assessment of the situation in terms of the hypotheses posed. In Bayesian analysis, we use Bayes rule and with belief functions we use Dempster's rule of combination. Although updating using Bayes rule is straightforward and poses no computational complexity problems, it does have difficulty dealing with

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[4] This is an upper bound based on the assumption that all propositions in the frame are focal elements (i.e., the all have support). The complexity is greatly reduced if the number of focal elements is small.

disconfirming evidence as discussed earlier. In addition, it requires that we assess the likelihood that the evidence just presented might occur conditioned on all possible hypotheses. These problems do not exist with the Dempster rule of combination. However, Dempster's rule has some problems of its own. The first is computational complexity as discussed earlier. However, if we are able to limit the focal elements to a rather small set, much computational complexity can avoided. The second problem deals with conflict. If two belief functions disagree totally ($\kappa = 1$), then one or the other must be discarded. However, we can measure the degree of conflict and establish a rejection rule applicable to the problem investigated.

Although neither method is perfect, and each method requires the subjective assessment of belief, the belief function methodology appears to be better suited to assessing the situation prevailing under conditions of uncertainty. Just the ability to assess belief in disjunctives makes it far superior to Bayesian methods.

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