Principal Component Analysis of the Fund for Peace Failed State Index: An Interdisciplinary Case Study

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ABSTRACT

Since its inception the Cornwallis Group has provided an interdisciplinary forum for the exchange of ideas on analytical approaches related to peace and stability issues. During the 2005 and 2006 Cornwallis Group meetings the Fund for Peace presented its Failed State Index based on twelve indicators of state failure. This paper presents the results of a study applying the mathematical technique of principal component analysis to both confirm the Fund for Peace's Failed State Index and to identify additional patterns and trends in the Fund for Peace data. This was done as a proof-of-concept in demonstrating the value of the Cornwallis Group in providing an interdisciplinary forum.

INTRODUCTION

The Cornwallis Group was set up to provide an interdisciplinary forum for the discussion of analytical approaches related to peace and stability issues. Each year efforts are made to encourage a wide range of participants from different sectors, such as defence, other governmental, academic and voluntary sectors, and from different academic disciplines. However, while the annual Cornwallis forum provides an opportunity for individuals from different sectors or disciplines to speak on peace and stability issues from their perspective, the task of integrating or synthesising this information with other perspectives has more or less been the task of individual Cornwallis participants.

During the 2005 and 2006 Cornwallis Group annual meetings, the Fund for Peace (Baker, 2006, 2007) presented their Failed State Index, which can be found on the internet at http://www.fundforpeace.org/web/. While the effort of the Fund for Peace is laudable, it was evident that sophisticated data analysis tools, such as principal component analysis, had the

potential to extract more information from the Index data than was currently being extracted. Therefore, as a proof-of-concept of the potential of the Cornwallis Group to provide a forum for interdisciplinary work by adding value through an integration of disciplinary perspectives as short study was conducted that applied principal component analysis to the publicly available Failed State Index data. In addition, the study would serve as partial review of the Fund for Peace approach.

THE FAILED STATE INDEX

Since 1996, the Fund for Peace has been developing its Conflict Assessment System Tool (CAST) which has been intended to serve as a conflict early warning tool and a system for conflict assessment. The tool consists of four primary steps (Baker, 2007), which are:

1. Evaluating twelve top conflict indicators;

2. Evaluating the "core five" state institutions of the political leadership, police and corrections, judiciary, the civil service and the military;

3. Examining surprises and idiosyncrasies; and

4. Mapping the conflict on a conceptual map that depicts a typical conflict life cycle.

Of particular relevance to this proof-of-concept analysis is the twelve conflict indicators. These indicators are each scored on a scale of one to ten with one being the best and ten the worst for an indicator of state failure. There are four social, two economic and six political / military indicators. The indicators are:

<u>Social Indicators</u>

1. Mounting demographic pressures;

2. Massive movement of refugees or internally displaced persons;

3. Legacy of vengeance – seeking group grievance or group paranoia;

4. Chronic and sustained human flight

<u>Economic Indicators</u>

5. Uneven economic development along group lines;

6. Sharp and / or severe economic decline;

<u>Political / Military Indicators</u>

7. Criminalization and / or de-legitimatization of state;
8. Progressive deterioration of public services;

9. Suspension / arbitrary application of the rule of law and widespread violation of human rights;

10. Security apparatus operates as "state within a state";

11. Rise of factionalized elites; and

12. Intervention of other states or external political actors.

Each nation's "failed state" score is the summation of these twelve indicators, giving scores between 12 and 120. The "failed state index" is the ranked list of nations by their failed state scores. Currently, the Fund for Peace classifies nations by groups according to scores of 12-30 for "sustainable", 31-60 for "moderate", 61-90 for "warning" and 91-120 for "alert".

While the "Failed State Index" is a significant achievement, the approach leaves several questions unanswered, which include:

1. Is the set of indicators complete or are there redundancies in the variables?

2. Is the simple summation of the scores an appropriate overall indication of the risk of state failure?

3. Is there a clear rationale for categorization of countries into groups?

4. Is there any additional information within the data that would be informative of the factors or processes related to state failure or recovery?

Assessing completeness (in item 1) would require eliciting judgements from experts in the field with regards to additional variables. In other words additional data would be required beyond the Fund for Peace data. This effort was beyond the scope this proof-of-concept study. However, within the limits of the existing Fund for Peace data it was feasible to assess redundancy. The focus of the study, therefore, was to apply mathematical analytical techniques appropriate to address the above questions, except for the caveat mentioned above, in order to demonstrate the value added of combining the insights from the political scientific and mathematical disciplines.

PRINCIPAL COMPONENT ANALYSIS (PCA)

Principal component analysis (PCA) (Jolliffe, 2002) is intended as a data reduction technique, by which a data set of a large number of often dependent variables are reduced to a data set based on a smaller number of uncorrelated (i.e. orthogonal) variables. The reduction is done so that as much of the variability in the original data set is retained as possible. This reduction in the dimensionality of the data set allows other techniques, including graphing, to explore data patterns and trends.

PCA achieves this transforming the data set to one based on a new set of uncorrelated variables, the principal components (PC), which are ordered so that the first few retain the

most variation in the original data set. Therefore, the 1st PC will be associated with the largest amount of data variation. PCA is a versatile technique that is applicable to many disciplines.

One method for determining this transformation is based on finding the eigenvalues and associated eigenvectors of the covariance matrix of the original data set. Suppose that the original data set, X, is an m by n matrix where m is the number of variables and n the number of observations. The steps of the method are:

1. Subtract the empirical mean (i.e., the mean based on the observations) of each variable from all observations of that variable;

2. Calculate the covariance matrix C, which will be of m by m dimensions;

3. Find the eigenvalues and eigenvectors of the covariance matrix C. Note that:

   $V^{-1} C V = D$,

   where V is the matrix of eigenvectors and D is a diagonal matrix with the eigenvalues along the matrix. While there are manual techniques for solving this equation, most software math packages such as Mathematica include functions for obtaining the eigenvalues and eigenvectors. Since a subset of these eigenvectors will eventually form the new basis set, it is important that they are of unit length. Most software math packages will provide unit eigenvectors;

4. Arrange the eigenvalues and eigenvectors in decreasing magnitude of the eigenvalues;

5. Compute the cumulative energy, which is related to the variance in the original data set, for each eigenvector and is simply the sum of the eigenvalues in order from the largest to the smallest;

6. Select a subset of the largest eigenvalues. This can be done by selecting the set of largest eigenvalues such that the cumulative energy exceeds a threshold value, such as 90%, of the total energy. There are alternative techniques for selecting eigenvalues, such as plotting the cumulative energy and choosing the eigenvalues up to a "knee" on the curve (i.e. up to the point of diminishing returns). This step is part of the "art" of PCA as selecting more eigenvalues and their associated eigenvectors includes more of the original variance but makes it more difficult to analyze data patterns and trends. Once the analyst has made a choice of eigenvalues, the set of associated unit eigenvectors (principal components) will form the new basis for the orthogonal subspace of reduced dimensionality;

7. Project the original data onto the new orthogonal subspace by:

   $Y = W^T X$,

where W is a matrix of l by m dimensions and l is the number of principal components chosen to define the orthogonal subspace. Y is a matrix of l by n dimensions that is the projection of the original data set of n observations onto the subspace defined by the eigenvectors of W.

The main assumptions of PCA are:

- Linearity through the change of basis;
- The mean and variance are sufficient statistics to describe the underlying probability distributions;
- The important dynamics are contained in the largest observed variances; and
- The principal components are orthogonal, which allows the use of linear algebraic techniques to find the PC.

A particular advantage of PCA is that it is a non-parametric method and, therefore, does not require any parameters or coefficients for the user to adjust. The answer is unique and independent of the user in that the defined subspace is unique. However, the direction of a principal component can be flipped without affecting the defined subspace.

The key disadvantage of PCA is that the PC are not observable variables but linear combinations of the original observable variables. This requires some "heuristic" judgment on the part of the user to interpret the PC. One way to do this is through a scatter plot of the elements of two PC. In other words, the eigenvector elements of each PC are treated as observations of two variables that are associated with the PC. Since the eigenvector elements are associated with the original variables, the plot gives the projection of the unit basis vector associated with each original variable onto the place defined by the two PC. Original variable basis vectors that are close to the origin in a given PC direction will not be associated with that PC while those further away will be highly correlated. Observing clusters of the indicators that mostly lie away from the origin and in the direction (or negative direction) of a PC can be used to define a "heuristic" characteristic of a PC. Note that this only allows a user to note which original variables are more or less associated with a PC. It does not allow a user to determine causality amongst variables associated with a PC.

The reader is encouraged to consult Jolliffe (2002) for further information on PCA.

APPLICATION OF PCA TO THE FAILED STATE INDEX

The PCA technique was applied to each annual (2005 and 2006) set of Fund for Peace data as well as to data for both years. The latter approach, having more "observations" in the data set is more robust and allows an assessment of changes for a given country from 2005 to 2006.

In terms of the specific questions posited above, the PCA method would address each as follows:

Question 1: Are there redundancies in the variables?

If one variable, according to country "observations", were completely dependent on another variable, this would be clearly indicated in the correlation matrix. An alternative approach is to compute the matrix rank (i.e. number of independent columns) of the correlation matrix. If this number is less than 12, the number of Failed State indicators in the data set, it would indicate redundancies. In this case the rank of annual or combined data sets, the matrix ranks was found to be 12. This indicates that, with dimensionality reduction technique like PCA, there will be a loss of information. However, assuming that the important dynamics are contained within the largest observed variations, PCA can still be used to reduce dimensionality.

Question 2: Is the simple summation of the scores an appropriate overall indication of the risk of state failure?

If the Fund for Peace Failed State Index approach is appropriate, the eigenvalue associated with the 1st PC will contain a significantly high percentage of the 'energy' in the correlation matrix and the constituent elements of that PC will have roughly equal values.

The Fund for Peace will be questionable if either of the following situations applies:

1. The 1st PC represents significantly high 'energy' but with over-weighted and under-weighted constituent elements; or

2. More than the 1st PC is necessary to account for a significant proportion of the variance or 'energy' in the data set.

These results can be found below in the section below in the discussion on the 1st PC for the annual data sets, but they confirm that the Fund for Peace approach is valid within the limits of the data set.

Question 3: Is there a clear rationale for categorization of countries into groups?

The Fund for Peace categorizes countries into four groups, which are "sustainable", "moderate", "warning" and "alert". While the Fund for Peace gives no rationale for this categorization, the Failed State index scores associated with these categories appear to be the maximum score of 120 divided into quartiles (i.e. under 30, 31-60, 61-90 and 91-120). While this has the benefit of simplicity there is no evidence that it is based on any dynamics in the data or on a clustering technique. Therefore, one cannot conclude that a "moderate" country with a score of 60 is at a significantly less risk than a "warning" country with a score of 61. The difference may not be significant.

While clustering techniques were not specifically applied to the data, the PCA method showed that not only was the overall Failed State score important, but so too was the indicator profile for a country. The indicator profile shows the variations across indicator values for a given country. This is discussed below in the section on changes in the data from 2005 to 2006.

Question 4: Is there any additional information within the data that would be informative of the factors or processes related to state failure or recovery

If the 1st PC does not have a significantly high enough cumulative 'energy' score, such as a score over 90%, then one should consider additional PC to the 1st PC to gain additional insight into the data dynamics. As it turned out the 1st PC was not sufficient to explain the data variance. The information in the additional PC were alluded to in the discussion on question 3 above; and related to the indicator profiles. A country's overall score and the structure of their indicator profiles had implications for how nations might fail or improve, at least, given that only two years of data were available, in terms of short term changes. Longer term dynamic trends would, of course, not be observed in such a short term data set.

RESULTS: ANNUAL FAILED STATE DATA

Unfortunately, the 2005 and 2006 data sets are not equivalent. There are only 76 countries in the 2005 data compared to a more complete set of 146 countries in the 2006 data set. In addition, the elements of the 2005 data set have not been selected randomly but only include what the Fund for Peace thought were the countries of greater concern (i.e. likely to have a higher Failed State score). Therefore, greater emphasis was put on the more complete 2006 data and the 2005 data set used as additional check on the results.

The eigenvalues for the 2006 data set are set out in Table 1. While the 1st PC (hereafter referred to as PC 1) does have a significant cumulative 'energy' value (78%), one would need to consider four PC to have a cumulative 'energy' score of 90% or greater. The projections of the indicators onto the PC 1 and PC 2, PC 1 and PC 3 and PC 2 and PC3 planes are given in Figures 1 to 3. PC 4 information is not shown graphically, but is discussed in the text below.

Rank	Eigenvalue	Cumulative Score	Cumulative Energy
1	50.87	50.87	78%
2	3.37	54.24	84%
3	2.75	56.99	88%
4	1.77	58.75	91%
5	1.37	60.12	93%
6	1.18	61.30	94%
7	0.98	62.28	96%
8	0.70	62.99	97%
9	0.63	63.61	98%
10	0.51	64.11	99%
11	0.48	64.59	99%
12	0.33	64.92	100%

Table 1: Eigenvalues and Cumulative Energy for the Fund for Peace 2006 data set.

An inspection of Figure 1 reveals information on both PC 1 and PC 2. As noted in Table 1, PC 1 has vast amount (78%) of the variance 'energy'. In addition, notice that the indicators are more or less equidistant from the PC 2 axis, which means that, with some minor variation, PC 1 is equivalent to the overall Failed State Index. Without testing the significance in the variations in the distance of the indicators from the PC 2 axis, it is reasonable to accept an equal weighting of the indicator values in determining the overall Failed State Index.

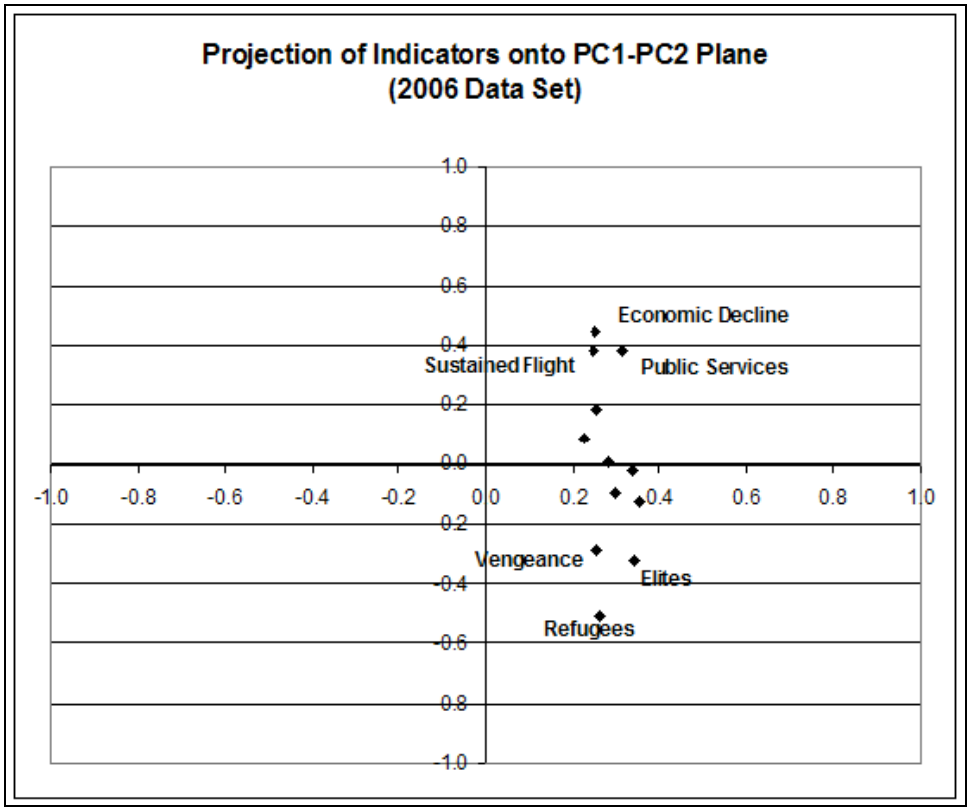

Figure 1: 2006 Failed State Index – 1st and 2nd Principal Components.

With regards to PC 2, Figure 1 shows that several indicators, which are clustered around the PC 1 axis, are not correlated with PC 2. However, PC 2 discriminates between countries that have correlated relatively higher scores for "chronic and sustained human flight", "sharp and / or severe economic decline" and "progressive deterioration of public services" with those that have correlated higher scores for the "rise of factionalized elites", "massive movement of refugees or internally displaced persons" and "legacy of vengeance – seeking group grievance or group paranoia." Interpreting this, one might say that there are countries with the sustained flight of people from poor quality of life and countries with refugees and displaced persons due to internal conflict. These are two categories that could be defined using PC 2 information.

Figure 2 shows the PC 1 and PC 3 plane. Again, we notice that the indicators are more or less equidistant from the PC 3 axis, confirming that PC 1 is roughly equivalent to the Fund for Peace Failed State Index. Considering PC 3, this component discriminates between countries correlated with regards to the "massive movement of refugees or internally displaced persons", "chronic and sustained human flight" and "intervention of other states or external political actors" and those correlated in terms of the "rise of factionalized elites", "criminalization and / or de-legitimatization of state", "suspension / arbitrary application of the rule of law and widespread violation of human rights" and "security apparatus operates as 'state within a state'".

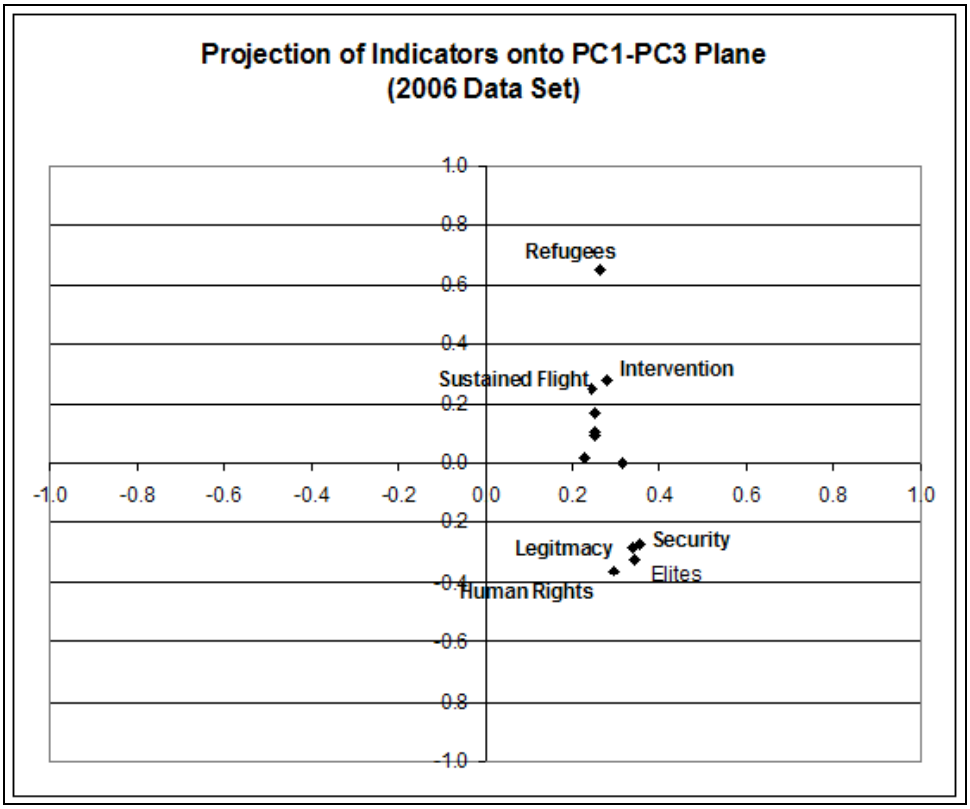

Figure 2: 2006 Failed State Index – 1st and 3rd Principal Components.

PC 4 is not shown graphically, but it discriminates between countries with correlated scores for "intervention of other states or external political actors", "uneven economic development along group lines" and "the "rise of factionalized elites" and those with correlated scores for "uneven economic development along group lines," "mounting demographic pressures", "massive movement of refugees or internally displaced persons" and ""progressive deterioration of public services." The interpretation here is less clear, which is not surprising as it is assumed that the less significant PC have a lower "signal to noise ratio."

It is particularly informative to consider the PC 2 - PC 3 plane, which is shown in Figure 3. Here we see that most of the indicators map onto three regions of that plane and that there does seem to be some commonality amongst the indicators mapped onto each region. These regions roughly relate to "quality of life", "governance" and "trans-border" indicators.

The indicator values for each country were projected into the PC subspace. Figure 4, in particular, shows projection of the countries onto the PC 2 - PC 3 plane, which had the three regions of "governance", "quality of life" and "trans-border" issues. Countries that map onto a region will have higher values for the indicators that correlate with that region. Those countries near the origin will roughly have little variance across the scores of their indicators, regardless of whether their Failed State Index score is high or low.

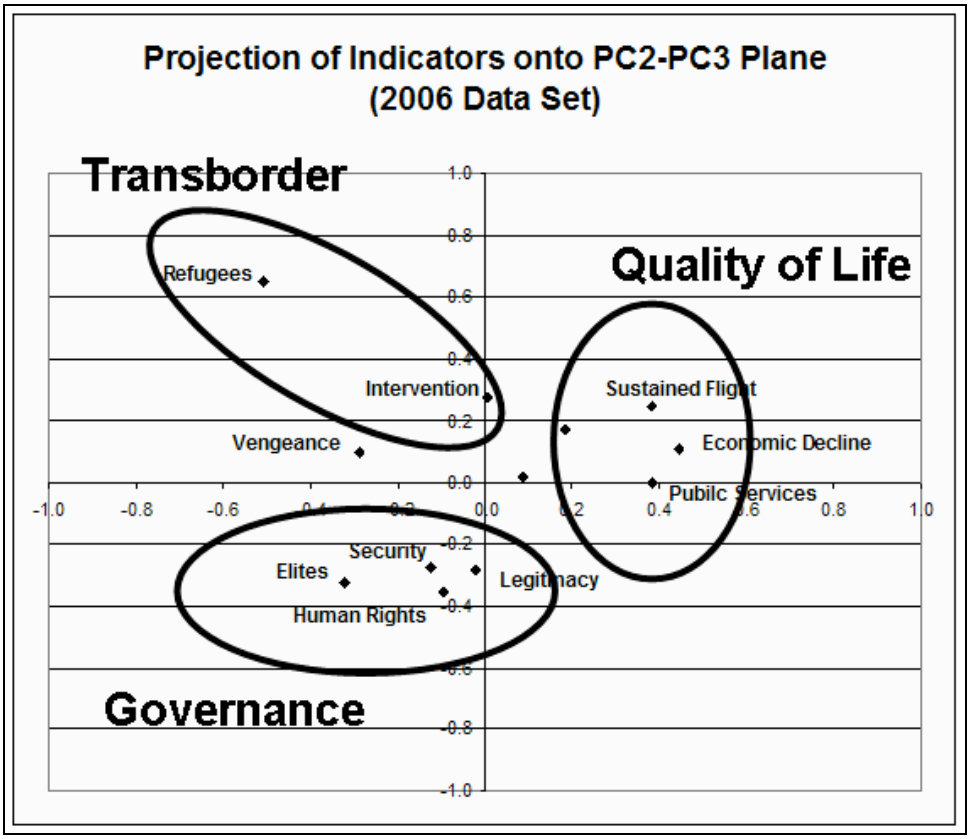

Figure 3: 2006 Failed State Index – 2nd and 3rd Principal Components.

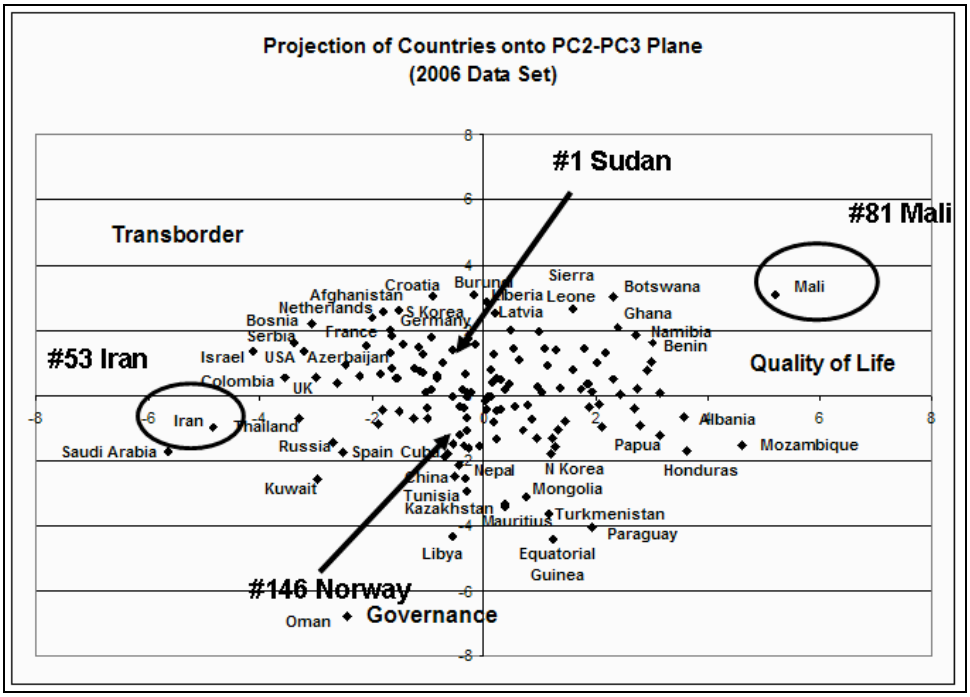

Figure 4: 2006 Failed State Index – Countries on the PC2 and PC3 Plane.

To illustrate profile variations, four countries are selected - the Sudan, Iran, Mali, and Norway. Their respective Failed State Index ranks were 1, 53, 81, and 146, respectively with 1 representing the worst score and 146 the best score. Their original indicator scores are

shown in Figure 5, whereby the indicators are arranged not according to their original order but according to their correlation with the three regions of the PC 2 - PC 3 plane. As can be seen, both Norway and the Sudan have a similar profile in that their indicator values are more or less equivalent across the indicators. The main difference is that the scores are smaller for Norway than for the Sudan. Note that the area contained within the boundaries of the indicator values on the spider plot are equivalent to the overall Failed State Index score.

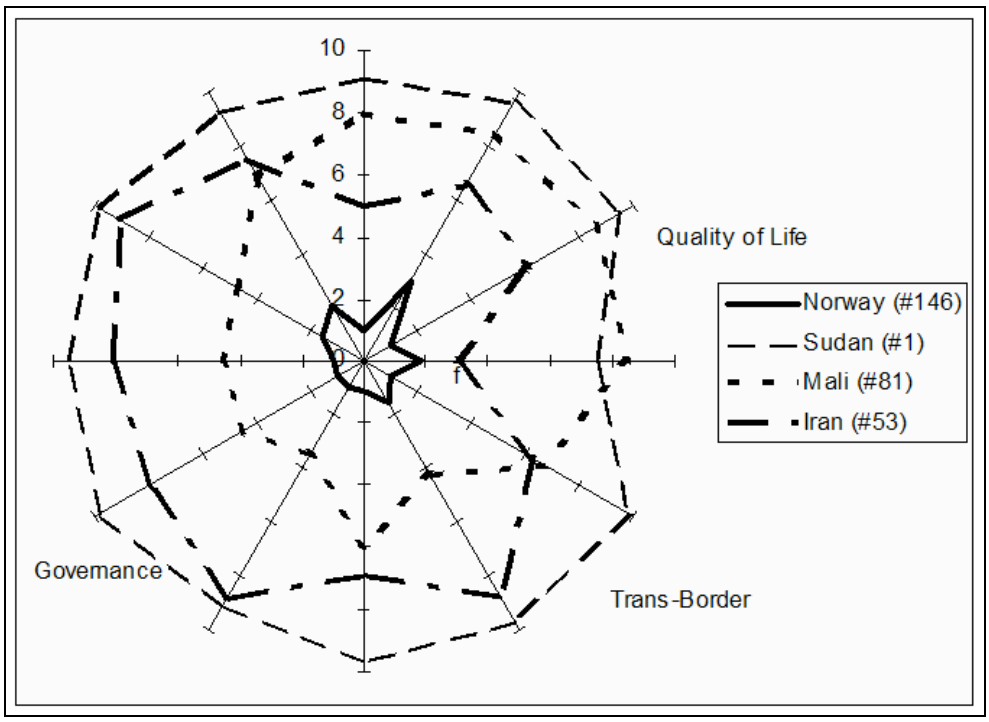

Figure 5: 2006 Failed State Index – Selected Countries.

By contrast Mail and Iran show marked differences across their indicator values. The profile for Mali bulges in the direction of "quality of life" indicators while the profile for Iran bulge sin the direction "governance" issues. Similar examination of countries from the different regions of the PC 2 - PC 3 plane show that roughly countries found in a given region have a similar profile if one subtracts out their main indicator value.

Similar analyses were conducted for the 2005 and the combined 2005 and 2006 data sets. In each case, PC 1 was found to be roughly equivalent to the Fund for Peace Failed State Index. Naturally, as one might expect since the 2005 data set excluded roughly half the countries with better Failed State Index scores, PC 1 did not have as high an 'energy' value for its associated eigenvalue. The results for the other PC were also much the same as found with the 2006 data set.

Figure 6 shows the projection of the indicators onto the PC 2 - PC 3 planes for the combined 2005 and 2006 data sets. The results are similar except that the regions are flipped. As mentioned earlier, while the subspace is unique, the directions of the principle components are not. So, while math software produced eigenvectors with different directions for combined data compared to the 2006 data alone, this "choice" was arbitrary and did not affect the defined subspace.

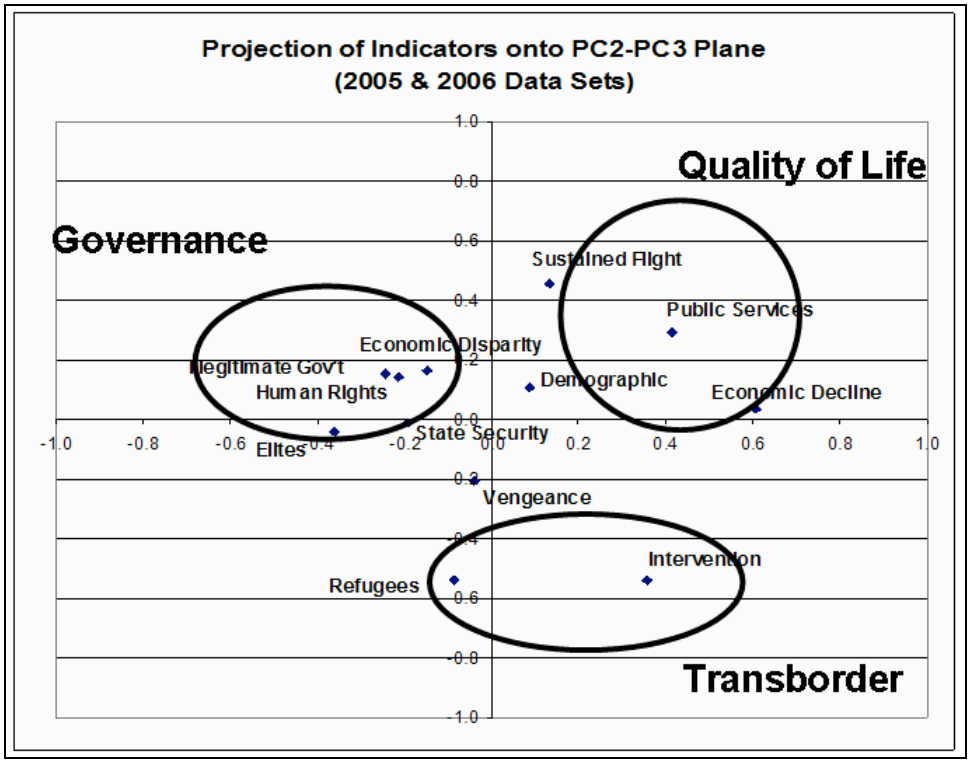

Figure 6: Combined 2005 and 2006 Failed State Indices – 2nd and 3rd Principal Components.

RESULTS: CHANGES FROM 2005 TO 2006

Analyzing annual data is useful for determining static measures. We need to examine data over several years to determine if there are common trends or "paths" by which countries worsen or improve with respect to their Failed State Index score. In this case only two years worth of data were available; and, even then, the 2005 data set had only half as many countries as the 2006 data set. Nonetheless, as a proof-of-concept study, this data proved sufficient to highlight some potential, interesting trends.

1. Worsening Countries: Figure 7 shows changes for the fifteen (15) countries that had the worst change in the Failed State Index from 2005 to 2006. The absolute values of the changes ranged from 4 to 14 points for Cameroon and Zimbabwe respectively. The relative changes were from 4% to 15%, which is about the same since these countries typically had 2005 Failed State scores 70 or higher. When the countries are grouped according to starting region and ending region, some interesting movements are noticed. Group 1 starts on the left side of the graph, in the region of countries with "governance" issues predominating and moves to the right. This group's starting Failed State Index scores range from about 72 to 80 and end up in the range of 82 to 90 with a change in their mean score from 77 to 87. Group 2 starts roughly in the middle, where Group 1 ended up, and moves to the "trans-border" region. The Failed State scores start in the range of 84 to 94 and ends up in the range of 88 to 109, with a change of mean score from 88 to 98. The final Group (3)

starts in the "trans-border" region and moves to the centre but at a worse "level" (i.e. Failed State score) than where Group 1 ended or Group 2 started. The initial scores for Group 3 range from 99 to 105 with a mean of 103 and end in the range of 106 to 112 with a mean of 108. These groups suggest that a more meaningful classification of countries might be associated with ranges of 70 to 80, 80 to 90 and 90 to 120 for stage 1 and 2 "warning" and "alert," respectively.

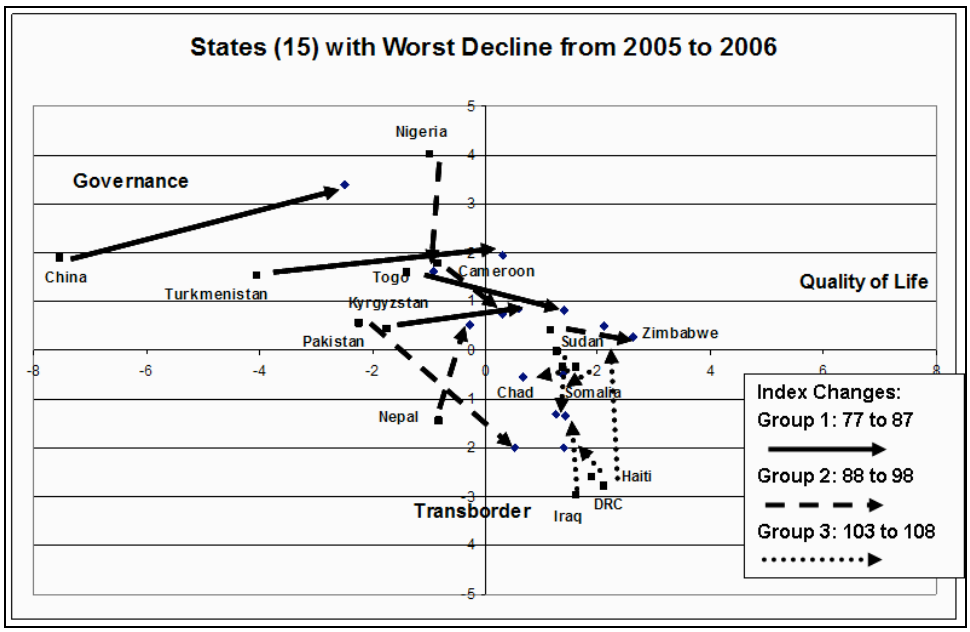

Figure 7: Changes on the PC2-PC3 plane for countries with the worst decline in PC1.

Note that generally the groups seem to be chained with Group 1 ending where Group 2 started and Group 2 ending where Group 3 started. The exception within Group 2 is Zimbabwe that made a huge jump from 95 to 109 and essentially ended up with Group 3. Figure 8 shows the spider chart of the average values for each of the Fund for Peace indicators arranged in accordance with the regions of the PC 2 and PC 3 plane. Group 1 shifted from the left ("governance" issues) and moved to the right ("quality of life" issues). However, while the major change was economic it was accompanied by deterioration in the state security apparatus, a "governance" issue. While causality cannot be certain, it is tempting to speculate whether the tightening of state security is a response or precursor to the economic decline. Figure 8 also illustrates the changes for Group 2. This group started near the centre of the PC 2 and PC 3 plane indicating that their conditions were more uniform across the indicators. When they worsened it was due to economic decline associated with movements of refugees or internally displaced persons and acts of group vengeance. Of course, with Group 3 there is a general degradation across all indicators.

It is speculative, but one can see, with these Group changes, a type of "death spiral" narrative whereby a country with "governance" issues is affected by economic decline perhaps causing dissatisfaction that is met by increased state controls and security. It would appear that this response is possibly counter-

productive in that it fails to halt economic decline and only increases dissatisfaction resulting in acts of vengeance, violence and the displacement of population, which in turn only further exacerbates the economic plight, governance problems, etc. The analysis in this paper is based on the internal structure of the data as defined by means and variances. Therefore, it is not suitable for validating narratives. This type of validation requires further research to find evidence of narrative details in real world case studies.

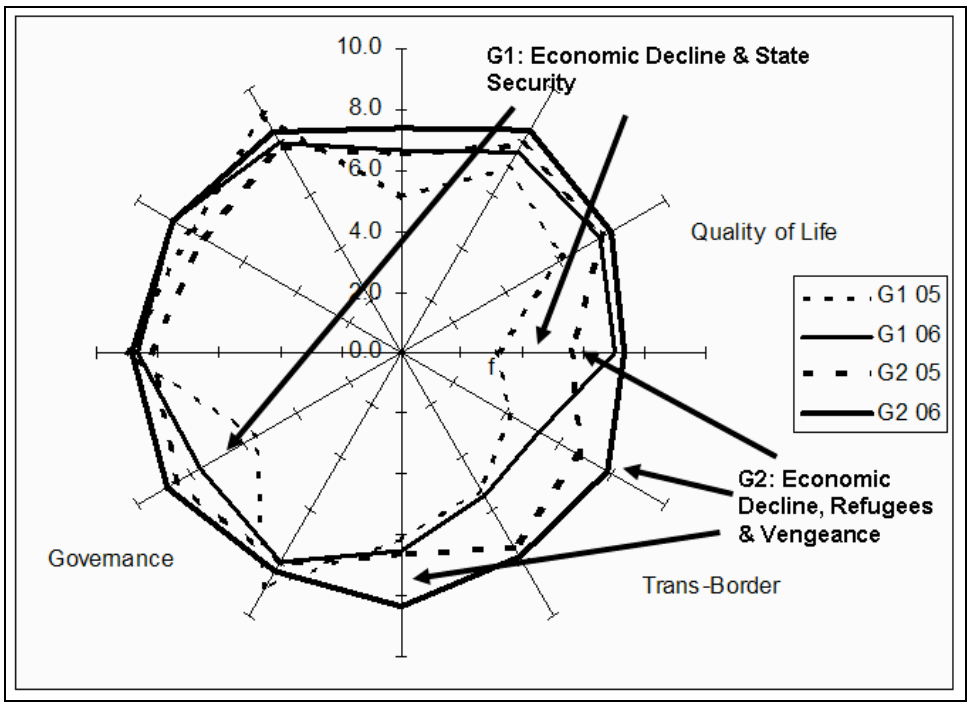

Figure 8: Indicator changes for the countries with the worst decline in PC1.

2. Improving Countries: Figure 9 shows the changes for ten countries that had the best improvements from 2005 to 2006. The improvements were between 10 and 20 Failed State Index points and the percentage changes ranged from 12% to 22%. The average initial starting Fail State Index score was 86. This is a slightly higher threshold for analysis than that chosen for "worsening" countries, but it was chosen to overlap the initial range of "worsening" countries, which was from 72 to 104 in order to see how the "improving" countries avoided what happened to the "worsening" ones.

   The "improving" countries were divided into two groups that corresponded to Groups 1 and 2 of the "worsening" countries. Like Group 1 of the "worsening" countries, Group 1 of the "improving" countries shifts from the left to the centre. However, as Figure 10 shows, that shift is not due to the economy declining faster than governance deteriorates. Rather it is due to improvements across the spectrum with significant improvements in both "governance" and "quality of life." A similar situation is shown for Group 2, which shifts in an upper right direction ending but ending somewhere in the middle. The direction shift here is for the same reasons as Group 1 but also includes some degree of resolution of refugee or internally displaced persons.

As mentioned, the narrative can only be speculative, but it would appear that the "improving" countries sought to address a broad range of issues, including "governance," and that one consequence of this was that they probably avoided the "quality of life" declines of the "worsening" countries. Again, real life world case study analysis is essential to validate potential narratives.

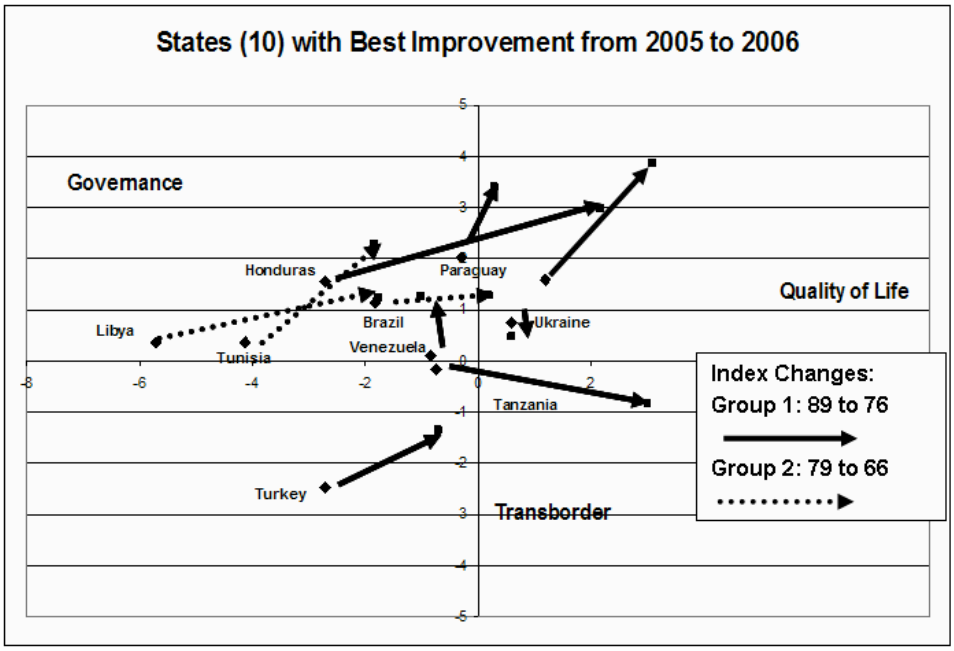

Figure 9: Changes on the PC2-PC3 plane for countries with the best improvement in PC1.

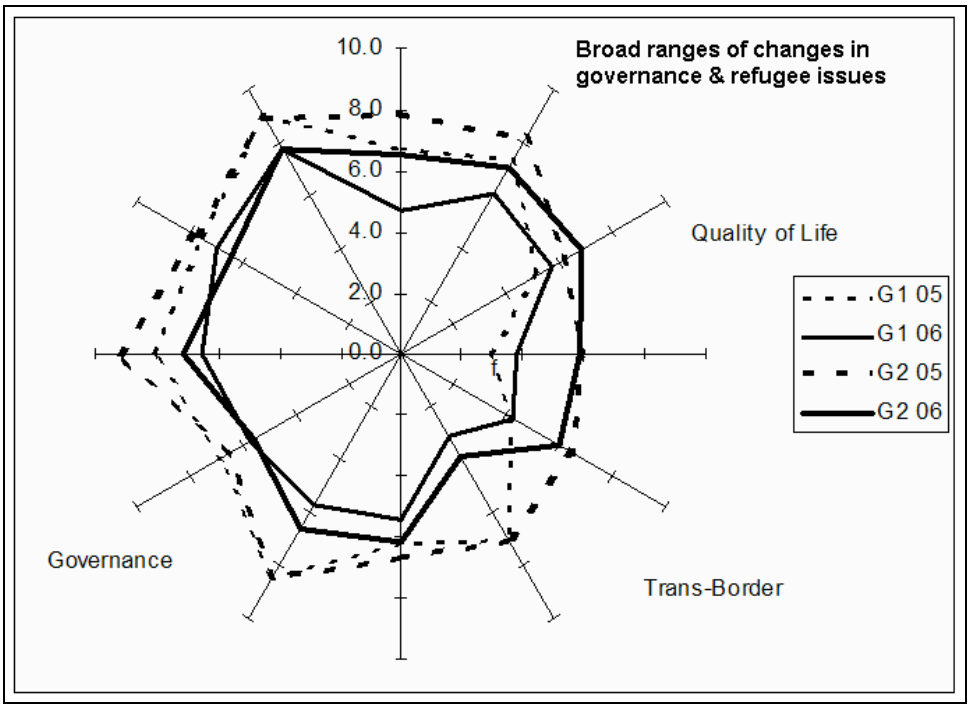

Figure 10: Indicator changes for the countries with the best improvement in PC1.

One trend that was noticeable by its absence was any significant decline from the right side of plane, the "quality of life" issues region. This would suggest that poverty alone is not a cause of state failure, at least not in the short term. Since the data spanned only one year, it is still possible that poverty is a major cause of state failure but that the decline is at a slower rate than declines due to bad governance. A data set spanning a longer period of time would be useful to understand dynamics related to poverty in the absence of major "governance" issues.

SUMMARY

This proof-of-concept achieved its objectives in validating the Fund for Peace's Failed State Index as being internally consistent with the indicator data sets. Whether or not the indicators and overall score are related to state failure requires a definition of what is meant by state failure and assessments from the real world that are independent of the indicator observations. PCA proved to be a useful tool even though the subsequent analysis of the reduced dimension data was fairly simple interpretation of graphs. Further analysis could be done using sophisticated mathematical methods for clustering, pattern recognition, dynamic flows, etc. In addition, the PCA method could be applied to alternative data sets of a similar nature but more extensive in terms of the number of years spanned. One well-known example is the highly respected Polity data series, which covers the years 1800 to the present. The data are available through contacts at the web site, http://www.cidcm.umd.edu/polity.

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