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Use of fractal-based approaches in the assessment of the Canadian recognized maritime picture

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Abstract

In order to understand the intrinsic structure of the geospatial distribution of vessel detections within the Canadian maritime regions, fractal-based methodologies were applied to detections from RADARSAT-2 and the Automatic Identification System (AIS). The original motivation was to determine which detections are likely to be false. In particular, symmetropy and spatial entropy (related to the fractal dimension) were applied to the data sets. These measures enable one to identify underlying non-linear (fractal) patterns in the spatial data related to emergent behaviour in non-linear systems. They were used in the past to distinguish between the different natures of various attack types in southern Afghanistan, as well as to investigate the fractal nature of the artificial combat in agent-based models. The results, including a comparison with artificially generated test data having known fractal characteristics, are discussed with the particular intent of identifying likely false positives, as well as identifying possible emergent patterns outside of the well-known main shipping routes; some of the implications for the conduct of surveillance operations are outlined as well.

Introduction

An understanding of the vessels that enter one's territorial waters or exclusive economic zone (EEZ) is vital in order to protect sovereign interests of any coastal country; it is no different for Canada. There are a variety of capabilities that can be employed to monitor shipping traffic on the approaches to Canada; the two discussed in this paper are and RADARSAT-2 (RS2) and

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the Automatic Identification System (AIS). RS2 is a commercial, sun-synchronous polar-orbiting Synthetic Aperture Radar (SAR) satellite providing imagery-based surveillance [1]. Since the RS2 satellite uses active sensing, detection of ships does not require voluntary or regulatory participation of the observed vessels. However, due to the nature of the orbit of the satellite, the area of surveillance is constrained on each orbit and the persistence is limited. On the other hand, AIS is a self-reporting system designed for enhancing safety of navigation at sea. AIS transponders are mandated by the United Nations Safety of Life at Sea (SOLAS) Convention [2] for all ships over 300 gross tonnage (GT), all passenger-carrying vessels, and it can also be used voluntarily by other vessels as well.

In order to exploit the surveillance of the Canadian approaches using these sensors, the following key questions can be posed:

- Are all detections corresponding to actual vessels, or are some of them false positives (i.e., apparent detections that do not correspond to actual vessels)?
- Are there any vessels exhibiting some kind of anomalous behaviour?

There are a variety of possible approaches to attempt to answer these questions. The approach proposed in this paper is based on the analysis of fractal (scaling) properties of the time-varying, spatial distribution of detections, employing a methodology related to previous work conducted by two of the authors on the deep nature of the geo-spatial distribution of violent events in Afghanistan [3]. There are three particular measures that are explored in this paper:

- Spatial entropy,
- Symmetropy, and
- Fractal dimension.

The approach is essentially pattern (trend) based; meaning that in order to observe an anomaly or false positive, one requires meaningful reference patterns and ways to detect changes in those patterns that correspond to activities of interest. In an environment where a complex mixture of order and randomness define the system dynamics, it can be a great challenge to appropriately characterize the interplay between the two, define the relevant trends or patterns, and then hone in on deviations of potential interest. It is hoped that this initial look at viewing traffic patterns in this way can be used to advantage in a maritime surveillance context, either through these measures or related ones that take advantage of any structural features found to reside in the data. Note that this treatment is not intended to constitute an exhaustive analysis of the detection data.

All three of the measures listed above are discussed in greater detail in the following section. This is followed by a brief summary of the data used in the analysis, and then a discussion of the results. Lastly, the findings are summarized and the implications for maritime surveillance are discussed.

This paper presents a first look at these measures in the maritime context and the feasibility of their use in a surveillance context, and it is not intended to be an exhaustive analysis of the detection data.

Spatial entropy, symmetropy, and fractal dimension

This section provides a brief description of the three measures employed in this paper. First, spatial entropy is described, then symmetropy and fractal dimension are discussed.

Spatial entropy is a specific form of Shannon (information) entropy $H = -\sum_i p_i \log p_i$ [3], where p_i is the probability of event *i*. This implementation is based on the original idea of evaluating the spatial distribution of ship detections relative to a regular grid covering an area of interest, which was first suggested by Ilachinski [4] for land combat applications. The resulting 'spatial entropy' is closely related to the fractal dimension (discussed below), when the latter is computed via the 'box counting' method. Compact, non-dispersed geometric patterns display low spatial entropy, whereas disorganized, spread-out patterns display high spatial entropy. The theoretical maximum for spatial entropy is $H_{max} = \log n$, where n would be the total number of points. In this paper, the value of spatial entropy was normalized so that the maximum value would be $H_{max} = 1$. Spatial entropy was employed by Ilachinski [4] to characterize the spatial distribution of soldiers on the battlefield, force concentration, and the degree of disorder. It has been also proposed as a possible measure to characterize crowd cohesion in crowd confrontation operations [5].

Symmetropy is another form of Shannon (information) entropy that measures the combined symmetry and entropy of a given two-dimensional intensity map [6]. It captures not only the spatial distribution, but the symmetry of the distribution as well. The definition of symmetropy utilizes a two-dimensional Walsh transform. Symmetropy components of a prospective pattern can be projected out with respect to four principal symmetries: vertical, horizontal, centro (also known as diagonal), and double (vertical plus horizontal). The strengths of the pattern symmetries relative to this basis are combined to provide an overall measure of symmetry in the pattern. In this paper, the symmetropy has been normalized to a maximum value of one,[2] corresponding to complete randomness. A drop in symmetropy corresponds to the detection of a dominant symmetry in the pattern. Zero is the minimum value, corresponding to an exact match to an element of the pattern basis.

The fractal dimension measures the minimum number of variables needed to specify a given pattern [4]. The dimension of fractal data sets is commonly approximated using the box-

[2] In reference [6], symmetropy ranges from 0 to 2.

counting (or capacity) dimension. It expresses the relationship between the size of a box, ε, and the minimum number, N(ε), of boxes needed to cover all of the ship detections. Generally, the dependence is a power law expression of the form $N(\varepsilon) = (L / \varepsilon)^{DF}$, where *DF* is the fractal dimension and *L* is the size of the area of interest. In general terms, the fractal dimension enables characterization of the clustering of the forces by one or the other side. In addition, it also captures the degree of distribution of the ships across the area of interest. The fractal dimension has been used in the context of combat modeling in a number of previous studies [4],[7]; it was used as one of the key coefficients linking temporal and spatial dynamics [7].

Complex Adaptive Systems Analysis (CASA) Software

The Complex Adaptive Systems Analysis (CASA) software employed in this study represents an early attempt by DRDC CORA to characterize both behaviour optimization and complexity awareness factors from the output of combat simulations conducted in the Map Aware Nonlinear Automata (MANA) model [8]. It was found that, overall, complexity awareness improved the quality of the mission outcome for the scenarios studied [9]. CASA is currently a research prototype programmed in C++ and Qt. One attrition-based measure (Carvalho-Rodriguez (CR) entropy [10]) is supported, as well as several spatial and vector-based measures.

Time series count-based measures appropriate for analyzing fractal-based point processes are also present. Although appropriate for dynamical analysis of a wide range of combat systems, the applicability of a given measure is highly scenario-specific – none are universally relevant and a particular measure is not guaranteed to provide any relevant information for a general situation of interest. One key determinant is how disorder unfolds in the system. Each entropy measure studied in some way rates the degree of disorder with respect to a reference notion about a particular aspect of the complex adaptive system that makes sense to observe. It is the selection of the reference views that determines what is found and what is not. As a scenario plays out, order and disorder appear as variations in computed measures, and are dominated by the rise and fall of projections onto the chosen reference patterns. For CR entropy, the reference pattern corresponds to phases of attrition. Spatial entropy [4] relates to patterns of detection concentration. Simple notions of dimension (e.g., fractal box-counting dimension) are extended via the generalized dimension, which examines multifractal properties and is closely related to Renyi entropy. For symmetropy [4], disorder is rated with respect to a spatial pattern symmetry basis. For the fractal dimension (closely related to order/disorder transitions), the reference patterns of interest relate to self-similarity and clustering. The Hurst coefficient [11] and the detrended fluctuation analysis (DFA) based self-similarity parameter [12],[13] rate vector deviations from what is essentially a random (long-term) walk. This proposed entropy based approach is not unique in application to maritime traffic analysis, as others have also applied measures of entropy to indicate the regularity of maritime traffic [14].

Systems displaying critical behaviour in the time series of events are typified by power-law scaling in the statistical distribution of events (event size – event frequency) and the presence of persistence or long-term memory in the time domain. The former is typically quantified through analysis of the spectral density, wavelets, and/or generalized dimensions [15] (also available in CASA), while the latter is often quantified via an alternative use of the aforementioned Hurst coefficient or some other form of fluctuation analysis (e.g., DFA) (see also [16] for potential pitfalls).

This list of measures studied is by no means complete. Disorder and fractal properties are not the only concepts of interest in the geo-spatial data analysis. Other prospective measures considered for future exploration include Grassberger-Crutchfield-Young's statistical complexity [17],[18] and total information and effective complexity [19], interval-based counterparts of the point process measures already mentioned (spectrum, wavelet variance, Hurst coefficient and self-similarity parameter), and cross-process measures such as wavelet covariance and the cross spectrum [15], to name a few.

RS2 and AIS data

For the analysis discussed herein, there were two types of data used: the operational RS2 ship detection dataset, which is provided by the Canadian Polar Epsilon (PE) project, and commercial satellite-based AIS (S-AIS) dataset (both illustrated in Figure 1, with individual detections as black dots). This dataset covers the entirety of 2014.

The Canadian RS2 SAR satellite is owned and operated by MacDonald Dettwiler and Associates (MDA). A commercial-government agreement between the Government of Canada and MDA is maintained, through which a number of departments purchase satellite data. The Department of National Defence's operational ship detection capability is delivered through the Polar Epsilon Project [1]. The two primary imaging modes used for ship detection are the Detection of Vessels, Wide swath Far incidence angle (DVWF) and the Ocean Surveillance, Very wide swath, Near incidence angle (OSVN) mode with 450km and 530km wide swaths, respectively. This imagery from RS2 is automatically processed by PE, and the ship detections are delivered to the operations centres. These global ship detections were logged for the period of 2014 and used in this analysis; the data for the area off the East and West Coast of Canada were used in this analysis.[3]

[3] As bounded by 30-62^o N of latitude and 30-80^o W of longitude for the East Coast and 30-62^o N of latitude and 120-170^o W of longitude for the West Coast.

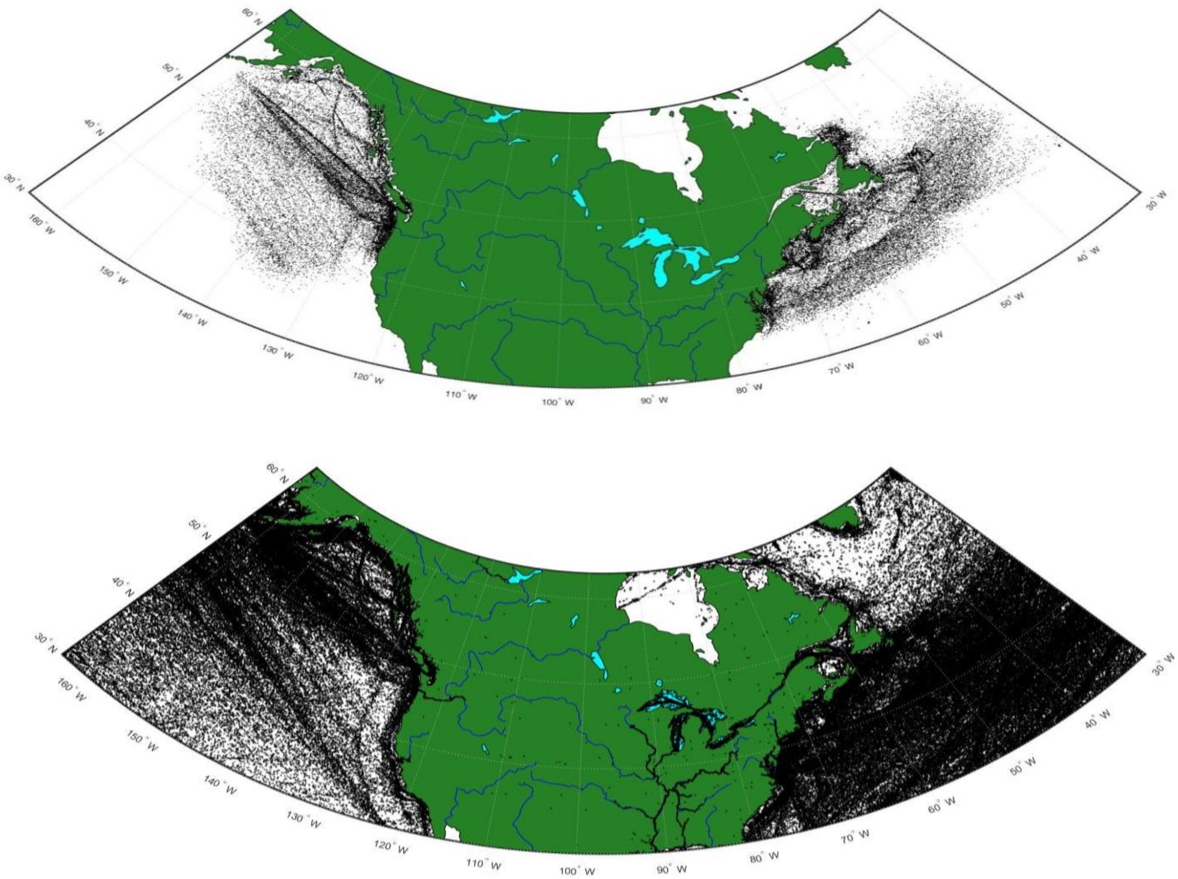


Figure 1: RS2 (top) and AIS (bottom) detections off of the Canadian coast for 2014.

SAR technology uses a moving active radar source to generate an image under the field of view. When operated in space, this field of view can be very large, which is one reason this technology is well suited for remote wide area surveillance. However, one of the challenges in space-based SAR is that the resolution of the radar image (i.e. the representative physical area represented by each image pixel) is inversely related to the size of the area being imaged. This trade-off in surveillance area versus resolution influences the size of target which can be detected. When observing ships on the ocean, special processing modes can be used to enhance the ability to detect ships.

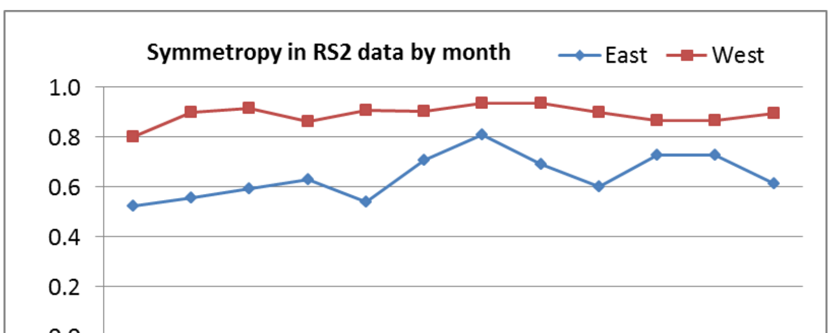
AIS is a radio-based transponder system which provides position and static identifying information for the purposes of maritime safety. AIS transponders transmit on Very High Frequency (VHF) radio at 161.975 Mhz and 162.025 Mhz. These transmitters have two primary classes: class A, with a minimum of 12 Watts power, and class B, restricted to a maximum of 2 Watts power. Although this system is designed for ship-to-ship operation, it is also possible to detect these radio transmissions from space. Even the low-power class B transmissions can be detected this way. The most recent AIS specifications now include optional special position message types which are designed specifically for satellite detection.

Since as early as 2008, the Government of Canada purchases commercial S-AIS from exactEarth® for the purposes of enhancing its Maritime Domain Awareness (MDA) [20]. This historical commercial data was collected over a year period (2014).

Results

There is a general limitation inherent in global measures such as the ones employed in this paper. They might require a significant shift in the geospatial distribution of the data to yield a noticeable magnitude of variations. In this paper, the dynamic properties of detections at the two Canadian coasts are compared using the coastal regions defined for this analysis. There is a fairly clear distinction between the East and West Coast visible in symmetropy and spatial entropy for both RS2 and AIS data as is discussed below.

The symmetropy in the RS2 data (Figure 2) shows greater randomness off the West Coast. There also seems to be a greater seasonal variability along the East Coast; notably, there is a peak in the summer months suggesting a transition toward more random observations. This is preceded and followed by fairly noticeable decreases. This effect could be possibly attributed to the fact that the commercial sailing (including fishing) season (that would tend to adhere to more regular traffic patterns) is likely to resume sooner after the winter slow down, and continue longer than the recreational sailing. On the other hand, the milder climate off the West Coast would sustain recreational sailing and sport fishing year around.



	1	2	3	4	5	6	7	8	9	10	11	12
East	0.52	0.56	0.59	0.63	0.54	0.71	0.81	0.69	0.60	0.73	0.73	0.61
West	0.80	0.90	0.91	0.86	0.91	0.90	0.94	0.94	0.90	0.87	0.87	0.90

Figure 2: Comparison of symmetropy in RS2 data between East and West coast.

Figure 3 shows the actual RS2 detections specifically for May and September 2014. The difference between the coasts is fairly obvious. For the West Coast, the detections are fairly uniformly distributed, with some increased density and generally symmetric patterns; this accounts for the somewhat higher value of the symmetropy compared to the East Coast. In the case of latter, there are clusters spanning several degrees of longitude; furthermore, the detection distribution is less symmetric, mostly as a result of complex coastal topology.

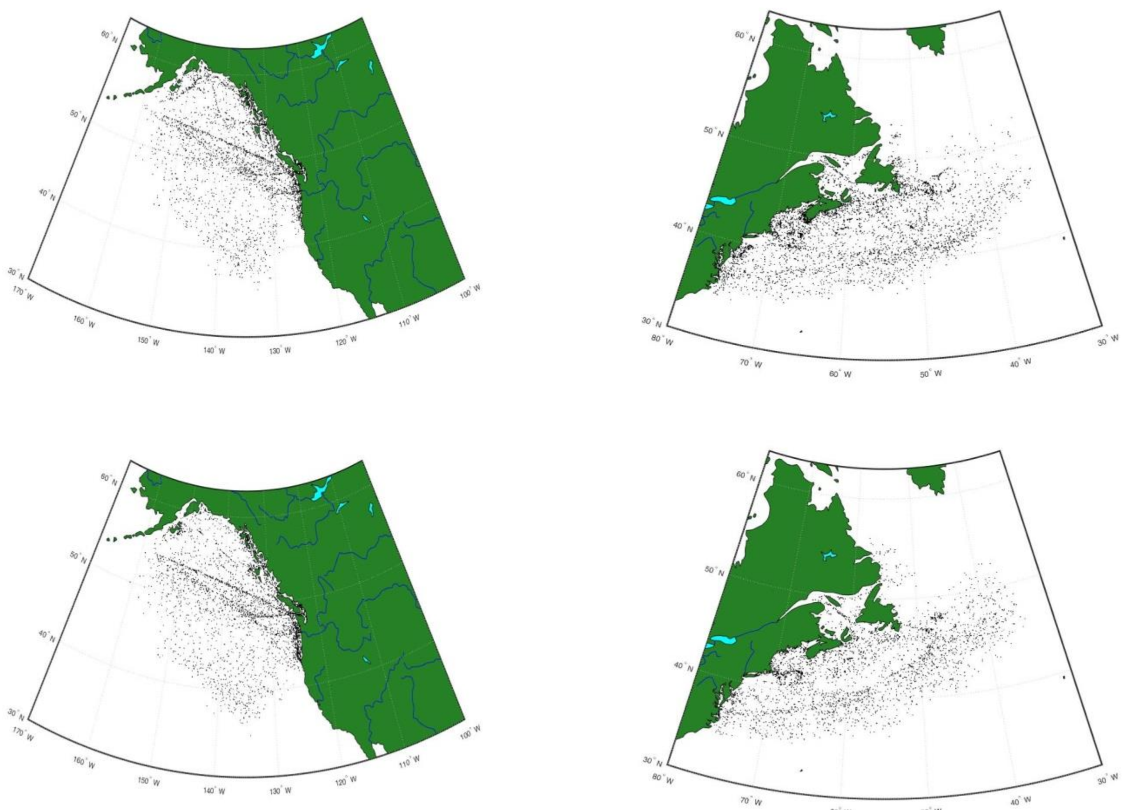
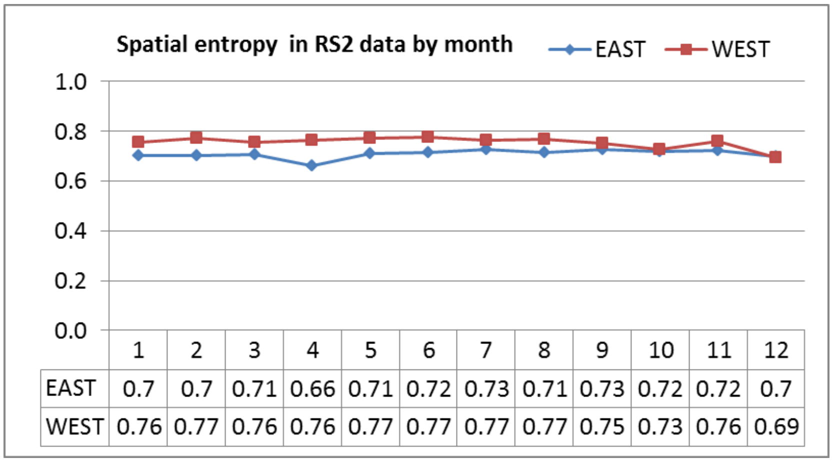


Figure 3: RS2 detection on the East (right) and West (left) Coast in May (top) and September (bottom).

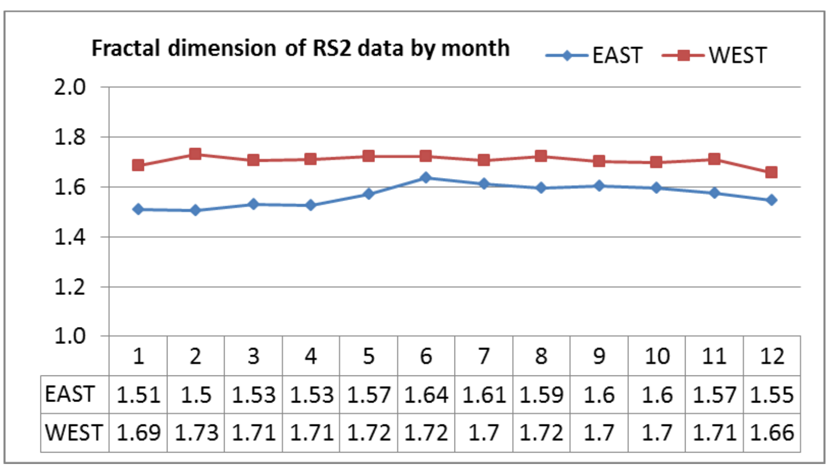
The spatial entropy (Figure 4) shows little month-to month variability for either coast. There is slight drop on the East Coast in April, and there is a similar drop in the West in October and December. Again, this hints toward a slight shift in towards more organized patterns (i.e., likely due to a greater influence from commercial traffic). However, the month-to-month variations are overall quite small. The value for spatial entropy being smaller than one (theoretical maximum that would correspond to uniformly distributed detections), indicates that the patterns are fairly non-random at both coasts.



	1	2	3	4	5	6	7	8	9	10	11	12
EAST	0.7	0.7	0.71	0.66	0.71	0.72	0.73	0.71	0.73	0.72	0.72	0.7
WEST	0.76	0.77	0.76	0.76	0.77	0.77	0.77	0.77	0.75	0.73	0.76	0.69

Figure 4: Comparison of spatial entropy in RS2 data between East and West Coast.

The results for fractal dimension of RS2 data (Figure 5) further reinforce the findings from the spatial entropy values. The value for the East Coast ranges between 1.5 and 1.6; for the West coast it is slightly higher, remaining steady around 1.7. This would be fairly consistent with the presence of patterns (and/or clusters) in the data. Slightly higher clustering off the East Coast could perhaps be attributed to the more complex geographical coastal structure of the East Coast compared to the West Coast (which, past Vancouver Island, is essentially an open ocean). An alternative explanation could be the presence of clusters of fishing fleets. Again, the month-to-month variations remain fairly small.

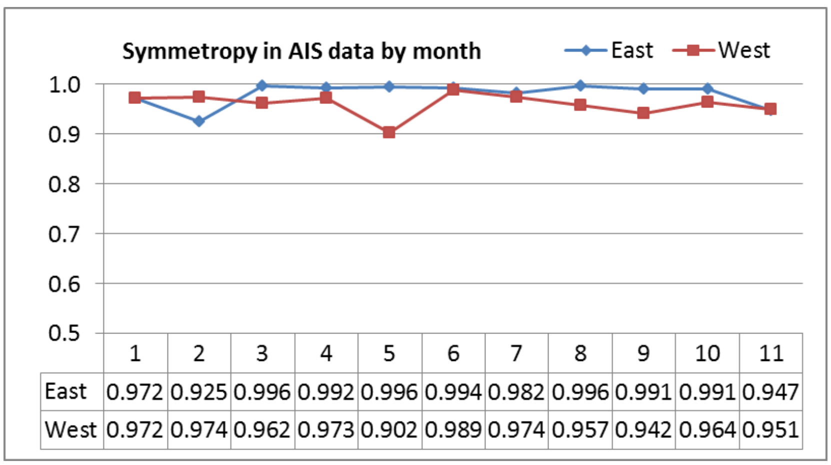


	1	2	3	4	5	6	7	8	9	10	11	12
EAST	1.51	1.5	1.53	1.53	1.57	1.64	1.61	1.59	1.6	1.6	1.57	1.55
WEST	1.69	1.73	1.71	1.71	1.72	1.72	1.7	1.72	1.7	1.7	1.71	1.66

Figure 5: Comparison of fractal dimension of RS2 detections between East and West Coast

Overall, the results for RS2 detections suggest that there is a non-random structure present in the detections. This might lead to a possibility of using the (non-) randomness of data subsets to determine a likelihood that a particular set may or may not correspond to false positives.

Figure 6 shows the results for symmetropy for the AIS data. Interestingly the result is somewhat reverse of the RS2 case. The symmetropy for the East Coast was generally higher, likely as a result of higher shipping density, staying very close to one. As is mentioned above, this might have been caused in part by the very large number of data points, leading to saturation of the distribution (due to a rounding error with the integer implementation in CASA). In addition, the West Coast featured a rather noticeable drop in May, suggesting a slight shift in the nature of the distribution.



	1	2	3	4	5	6	7	8	9	10	11
East	0.972	0.925	0.996	0.992	0.996	0.994	0.982	0.996	0.991	0.991	0.947
West	0.972	0.974	0.962	0.973	0.902	0.989	0.974	0.957	0.942	0.964	0.951

Figure 6: Comparison of symmetropy in AIS data between East and West Coast

The data for the West Coast is examined in further detail, utilizing April as a comparator, and the geo-spatial distribution of the AIS data is shown in Figure 7. From this figure, it can be seen that the April data presents a fairly random background distribution, with the increased detection density due to some major shipping routes being visible. The May data show clustering along the coasts and in the oval band indicated on the figure. It is postulated that the coastal clustering is due to the opening of seasonal fishing, and the band corresponds to the great circle routes of a variety of shipping routes that are traversed as the sea state in the Pacific Ocean calms down from the winter storms. Once the initial 'spike' in shipping is over in May, the pattern returns to normal, increasing steadily until September and then dropping off again in the winter.

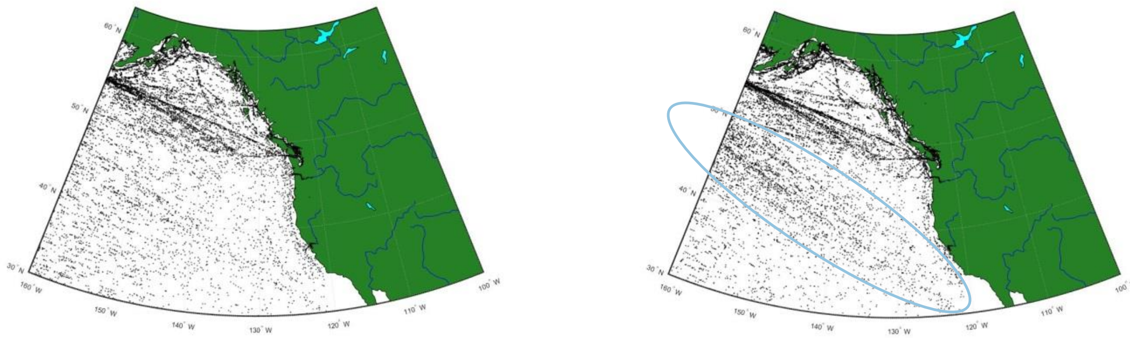
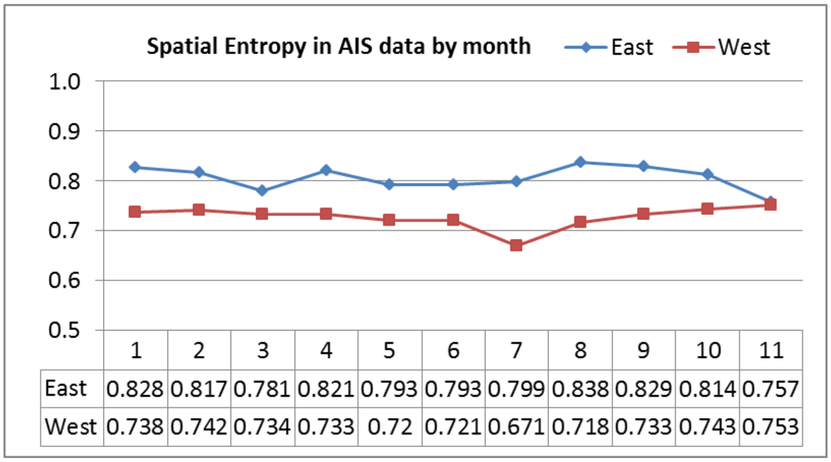


Figure 7: AIS detection on the West Coast for April (right) and May (left).

The spatial entropy (Figure 8) strongly suggests that there are underlying patterns (non-randomness) in the data in both East and West. The spatial entropy was lower for the West Coast, suggesting a greater degree of internal organization. Interestingly, there seems to be an offset between the changes in spatial entropy and symmetropy. For example, the May drop in West Coast symmetropy is not reflected in spatial entropy, but there is a drop in the latter in July that has no correspondence in the former. Similarly, the East Coast shows a drop in symmetropy in February, while in spatial entropy similar drops occur in March. Because symmetropy is related to phase transitions in a system [4], while spatial entropy measures the level of organization in general, it is possible that there are some fundamental drivers[4] that affect the latter without causing actual global reconfiguration (i.e., phase transition).

[4] These drives can include major global influences such as climate, including seasonal winds, ice coverage, or presence of storms, presence of major shipping routes, or more localized factors such changes in a port capacity, fishing seasons, or major recreational sailing events.



	1	2	3	4	5	6	7	8	9	10	11
East	0.828	0.817	0.781	0.821	0.793	0.793	0.799	0.838	0.829	0.814	0.757
West	0.738	0.742	0.734	0.733	0.72	0.721	0.671	0.718	0.733	0.743	0.753

Figure 8: Comparison of spatial entropy in AIS data between East and West coast.

It was noted there was an data saturation issue related to the implementation of CASA[5]; the symmetropy and spatial entropy would converge to higher values (i.e., suggesting a more random distribution than it actually was). This is reflected, for example, by the fact that the symmetropy values were much closer to one than those for the RS2 data.

Unlike for the RS2 detections, for the AIS the fractal dimension (Figure 9) converges to values close to two. This suggests near-random distribution uniformly covering the entire investigated area. However, this seems to be in contradiction to the apparent denser areas observed in Figure 7. As is discussed above for symmetropy, this appearance of uniform distribution in fractal dimension may have been caused by the data saturation caused by the way the calculations were implemented in CASA (i.e., the need to translate the actual locations to an integer grid).

[5] The actual locations were translated to an integer grid (with finite number of locations). Thus as the number of detections increased, so did the likelihood of each location being occupied by at least one translated location.

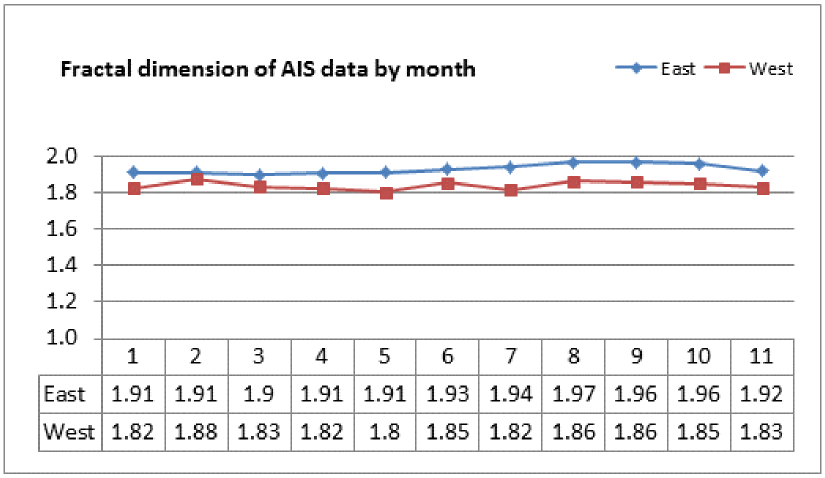


Figure 9: Comparison of fractal dimension in AIS data between East and West coast.

Summary and Conclusions

Overall, the results suggest that the maritime detection data off coast of Canada exhibit an intrinsic geo-spatial structure reflecting non-random patterns (possibly including shipping routes and fishing areas). This has an implication for the analysis of data subsets; for example, a lack of any structure in a particular subset of the data, i.e., random distribution of the data, might imply that the particular subset contains false positives (i.e., detections that do not correspond to actual vessels). A caveat needs to be repeated here. The global measures, such as symmetropy and spatial entropy employed in this paper, have one general limitation. They are likely to require a significant shift in the geospatial distribution of the data to produce visible variations. Hence the actual value of the methodology, apart from the temporal analysis of global patterns as has been done in this paper, might be in the comparison of large subsets of the detections (e.g., detections corresponding to a particular vessel type) with each other and with the overall detection distribution.

The symmetropy identified some month-to-month variability in the detections for both RS2 and AIS. However, the results from the two data sources (RS2 and AIS) gave almost opposite trends. This could be either due to problems with data saturation in CASA or perhaps relates to the detection methodology[6]. This somewhat limits the value of this measure with respect to the comparison of dynamic/fractal properties across sensors, but it would have little impact on comparing subsets of the detections collected by the same sensor over the same time.

[6] For example, RS2 may be better at detecting larger sized vessels vice smaller, whereas AIS may be influenced by the AIS transponder power irrespective of vessel size.

The variations in the fractal structure could have been caused by seasonal changes in shipping traffic (e.g., cruise ship season beginning in May), fishing seasons, weather (such as possible presence of ice in certain areas), etc. Overall, the results suggest that there are non-random patterns present in the detections; this, in possible combination with other indicators, might potentially provide means for the identification of subsets that do not conform to overall trends. These non-conformal subsets could be then either discarded from further analysis (if, e.g., they are likely to be false positives), or alternatively they can be subjected to closer scrutiny (if there are valid detections deviating from these patterns). However, before any attempts to employ any of these measures for pattern recognition in subsets can be made further validation of the actual implementation of these measures is required; in particular the granularity of the grid in CASA must be increased to accommodate large data sets. Follow-up steps might include removing zeroth-order pattern (e.g., main shipping routes) and analyzing properties of the background detection distribution. This would be akin to the analysis conducted on the Afghanistan violence data; in the latter case the data had to be transformed and the incident ring along the Ring Road were removed in order to assess the higher-order properties of the incident distribution [3].

Future work on this topic includes further research into validating the observed seasonal effects with known commercial and recreational schedules, as well as major weather trends. Furthermore, further research will be required to evaluate the utility of spatial entropy to evaluate the behaviour of ships which could potentially be identified as being normal or abnormal based on the nature of their motion given the area and season. While, as is mentioned above, the global measures would unlikely provide sufficient resolution to assess changes caused by a single vessel (or a few vessels), they could be used to develop a probability distribution maps for what should be considered typical behaviour. Hence, the insights provided by these measures would be probabilistic, and would have to be used in conjuncture with other criteria.

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