Validating Lanchester models: the first ~~60~~ 120 years

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Abstract

Computer-based combat models are some of our most important decision support tools, and at the heart of these lie Lanchester attrition models, implicitly if not explicitly. One enduring problem has resurfaced recently: Lanchester and later theorists defined a *family* of attrition models, and it is by no means clear which of these models should be used in which circumstances.

The paper traces our attempts to validate Lanchester's models using quantitative historical analysis (HA), starting in 1954, two milestone papers from 1987, and later studies that have reinforced their conclusions, using several HA methods.

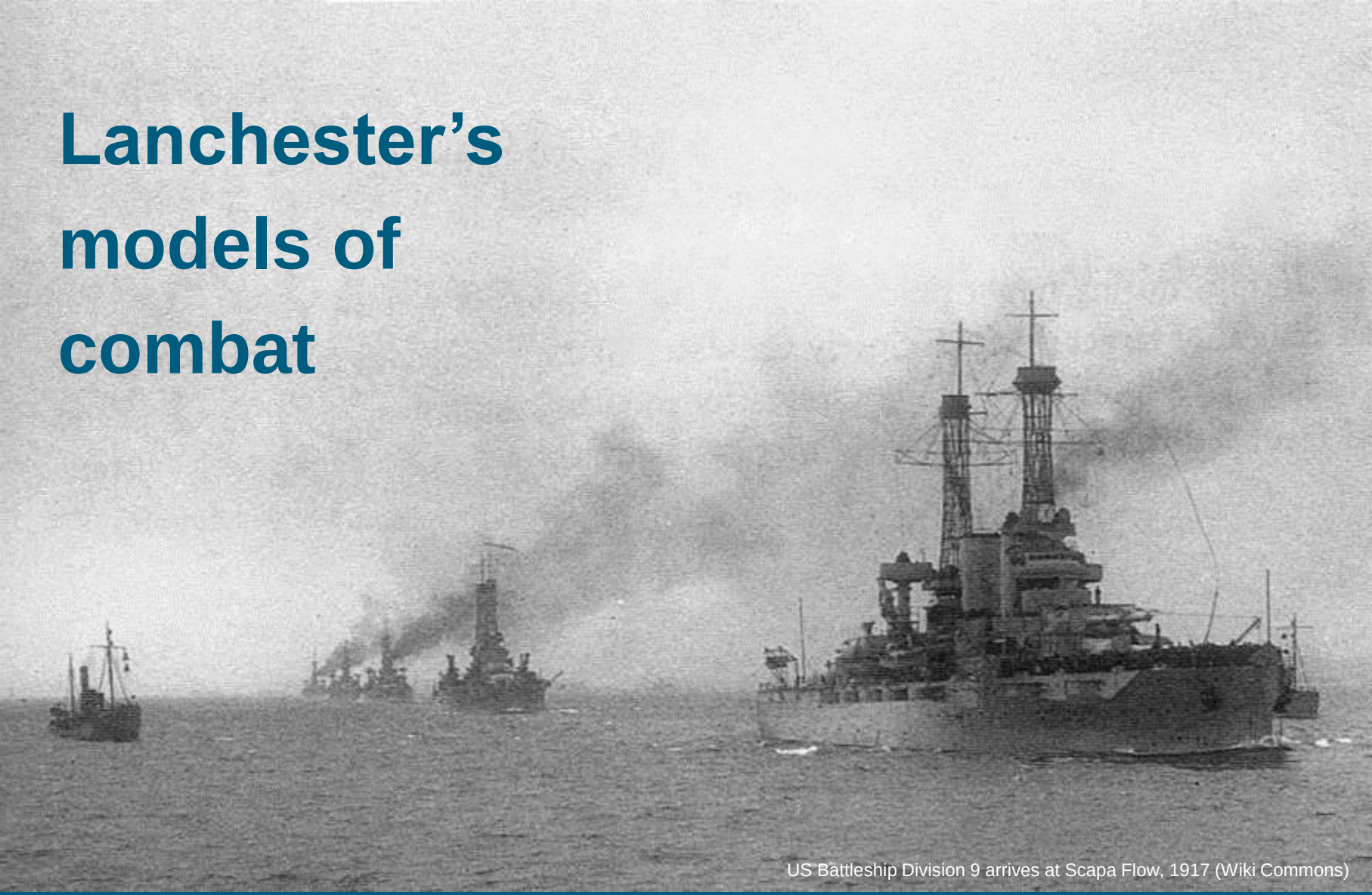
It then considers how these mechanisms might operate in reality. It is important that we understand these mechanisms quantitatively in order to implement them in computerised models that do not use Lanchester equations explicitly, and that we devise a method by which historical battles can be compared directly with computer model output. We also need to understand the impact on the type of input data required by the revised models.

Finally, it will consider the importance of getting this right, and the potential effects on the age-old debate of quantity versus quality, numbers of troops versus technology and training, which lie at the centre of defence planning.

Topics

- Lanchester's models of combat
- Historical battle analysis
- First attempts to validate Lanchester's equations
- The case is proven in 1987?
- A recent Dstl study
- Implications for combat models
- Conclusions and questions

Lanchester's models of combat



US Battleship Division 9 arrives at Scapa Flow, 1917 (Wiki Commons)

First proposed by …

- Lieutenant J.V. Chase USN, in 1902*
  - talked through a ‘Lanchester square effect’ in prose
- Commander Bradley Fiske USN, 1905*
  - described the ‘Chase effect’ using time-stepped tables
- Both intended them to be *naval* models

* See Hausken & Moxnes (2000)

Frederick Lanchester, 1914 & 1916

- Formulated two sets of differential equations
- Lanchester's 'square law' for individual fire

$$dB/dt = -kb.R, \quad dR/dt = -kr.B$$

  - force effectiveness = individual effectiveness × n^2
- Lanchester's 'linear law' for area fire

$$dB/dt = -kb.R.B, \quad dR/dt = -kr.B.R$$

  - force effectiveness = individual effectiveness × n
- Arrived at independently by Mikhail Osipov in 1915

A *family* of attrition models

- Various theorists have expanded Lanchester's laws
- Mixed law: Smith (1965)
  - sum components of linear and square laws
- Logarithmic law: Peterson (1967)

$$dB/dt = -kb.B, \quad dR/dt = -kr.R$$

  - loss rates determined by *own* numbers only, e.g. DNBIs, FF
- General law: Bracken (1995)

$$dB/dt = -kb.R^p.B^q, \quad dR/dt = -kr.B^p.R^q$$

Maths for its own sake?

- Brown (1963) … add your own full stops …

STATEMENT OF THE PROBLEM

Two FORCES, I and II, engage in combat It is assumed that there are no replacements, so that the only changes in the strengths of the forces are decrements As the struggle goes on, the numbers of survivors on the two forces will tend to diminish The force that first reaches zero is said to lose the battle, and the other force is said to win

- Taylor (1980) presented >1400 pages of maths …
  - hardly mentioned a single historical battle!

Historical battle analysis

The battle of Mars-la-Tour in 1870 (Wiki Commons)

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Historical analysis

“The use of mathematical, statistical, qualitative and other forms of analysis to explore and understand historical military engagements, operations, campaigns and conflicts.”

Why do quantitative HA?

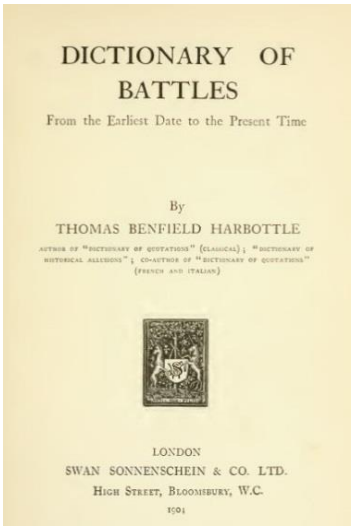
- Foundations of understanding battle
- Rapid analysis and insight for policy decisions
- Validating battle models
- Analysing human factors
  - e.g. deception, surprise, surrendering, value of training
  - not easily modelled in a computer
- Gathering otherwise hard-to-source data
  - e.g. formation rates of advance – multi-variable inputs

Earliest roots of quantitative HA

- Aimed to understand battle *quantitatively*
  - following Clausewitz, Jomini, and Du Picq's qualitative work
- Otto Berndt *'Die Zahl im Krieg'*, Vienna, 1897
  - catalogued 96 European battles and sieges, 1741 to date
  - concluded that winning or losing generates casualties …
    - not the other way round!
    - Most casualties inflicted *after* a military decision reached

We need more data …

- Thomas Harbottle's *'Dictionary of Battles'*, 1901
  - qualitative data on ≈1700 battles from antiquity to date
- Gaston Bodart's *'Militar-historisches Kriegs-Lexikon'* , Vienna, 1908
  - statistics on ≈1500 land battles, 1618–1901


DICTIONARY OF BATTLES

From the Earliest Date to the Present Time

By

THOMAS BENFIELD HARBOTTLE

LONDON
SWAN SONNENSCHEIN & CO. LTD.
HIGH STREET, BLOOMSBURY, W.C.
1901

First attempts to validate Lanchester's equations

US Marines raising the flag on Iwo Jima (Joe Rosenthal, AP)

Supporting first computer models

- Engel (1954) tested *vs*. Iwo Jima in 1945
  - used USMC daily loss rates, consistent with square law
  - supported independently by Samz (1972)
  - but special tactical circumstances later questioned
- Willard (1962) analysed land battles 1618–1905
  - 1493 European battles from Bodart's *Kriegs-Lexikon*
  - victory independent of FR if FR<4:1 or 5:1
  - *"Lanchester's square law is the poorest among poor alternative choices"*

Anderson *et al*., 1972

- Attempted to model battle of Cambrai, 1917
  - British combined arms attack including early tanks
  - exploiting availability of detailed British casualty rates
- Criticisms: one-sided approach
  - no comparable German loss data by time were available
- Didn’t really nail the Lanchester issues

More qualitative evidence

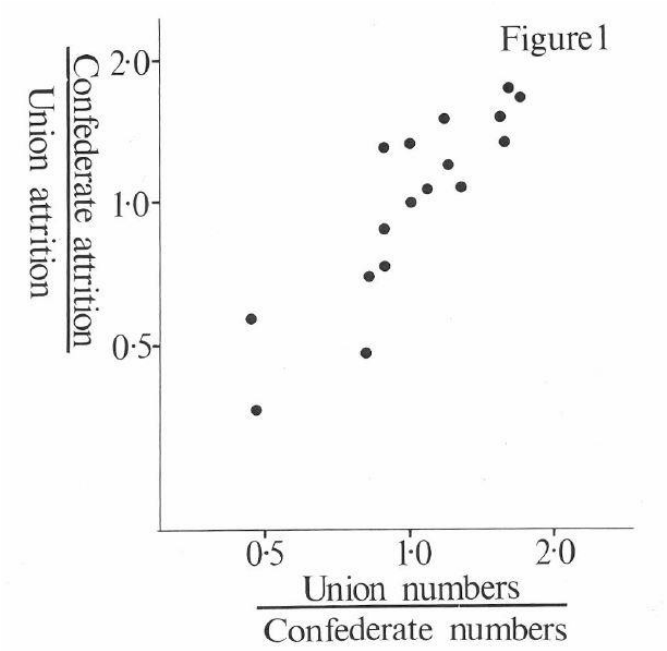
- US GAO report PAD-80-21 (1980)
  - *“From a scientific point of view, the present understanding of war is in a relatively primitive state.”*
- Handel (1981) discussed quality vs. quantity
  - both in materiel and manpower
  - on basis of recent military experience (1967, 1973, etc.)
  - challenged Lanchester’s emphasis on numbers

Pizer, 1984

- Instrumented field exercise *vs*. computer model
  - Soviet-doctrine tank battalion attacking ATGWs and tanks
- Consistent with Lanchester square law, but …
- Small numbers of defenders and few reps
- All weapons were DF, ‘point-to-point’
  - fitting Lanchester square law is no surprise!

Kirkpatrick, 1985

- Analysed 17 ACW meeting engagements, 1861–65
- Higher FR lowers LER
  - good correlation, $r^2 = 0.89$
- Closer to linear law
  - FR therefore less important
  - quality equally important



Case proven in 1987?

6 Bn Green Howards landing on Sword Beach, 6 June 1944 (IWM)

John Lepingwell, 1987

- Re-examined the Lanchester and HA evidence
- Concluded that the linear law better fitted the data
  - explanation: *most* weapons are area weapons
- So why didn't everyone change their models?
  - a lot of scepticism about this 'new' HA among simulationists
    - particularly at RARDE Fort Halstead!
  - organisational inertia, sunk costs in existing models …
    - all of which used Lanchester square for infantry combat

David Rowland, 1987

- Plotted force ratio against attack casualties/defender
- Using more attackers results in *more* casualties

D. Rowland—Assessment of Combat Degradation

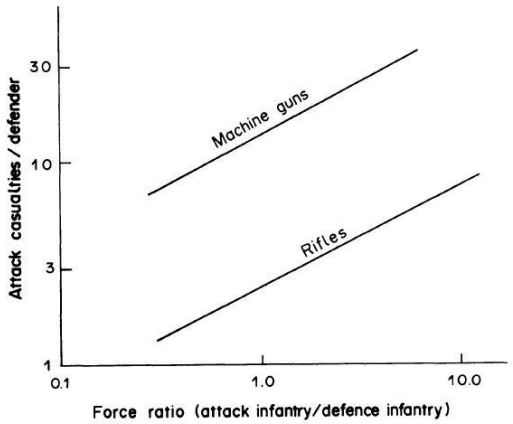
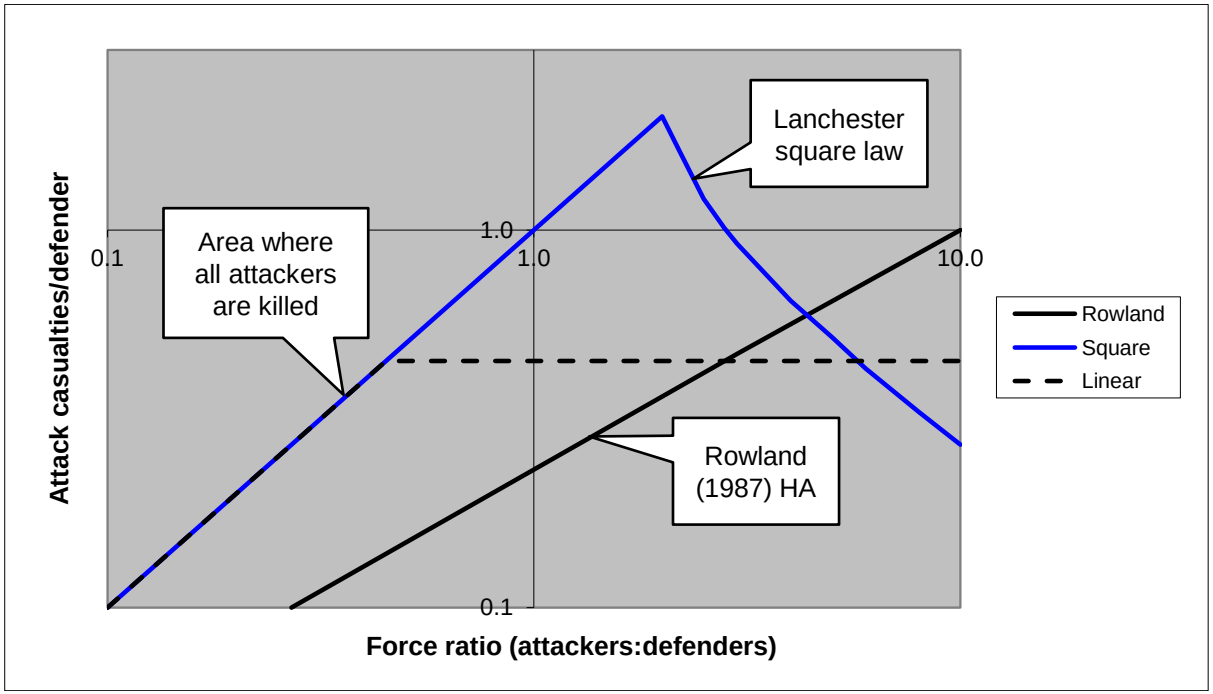


FIG. 1. *Variation of attack casualties/defence weapon with force ratio from interactive trials data.*

Rowland *vs*. Lanchester

- Force ratio against attack casualties per defender



But the debate rumbled on …

- Focussed on Kursk in 1943, and Ardennes in 1944
  - enabled by detailed historical research in 1990s
- Bracken (1995) fitted Ardennes data to the linear law …
- Fricker (1998) fitted same battle to log law!
- Lucas & Dinges (2004) fitted Kursk data to Bracken's 'general' Lanchester law
  - but linear and log laws fitted better than the square law
  - *best fit varies with the phase of battle*

We need a bigger database!

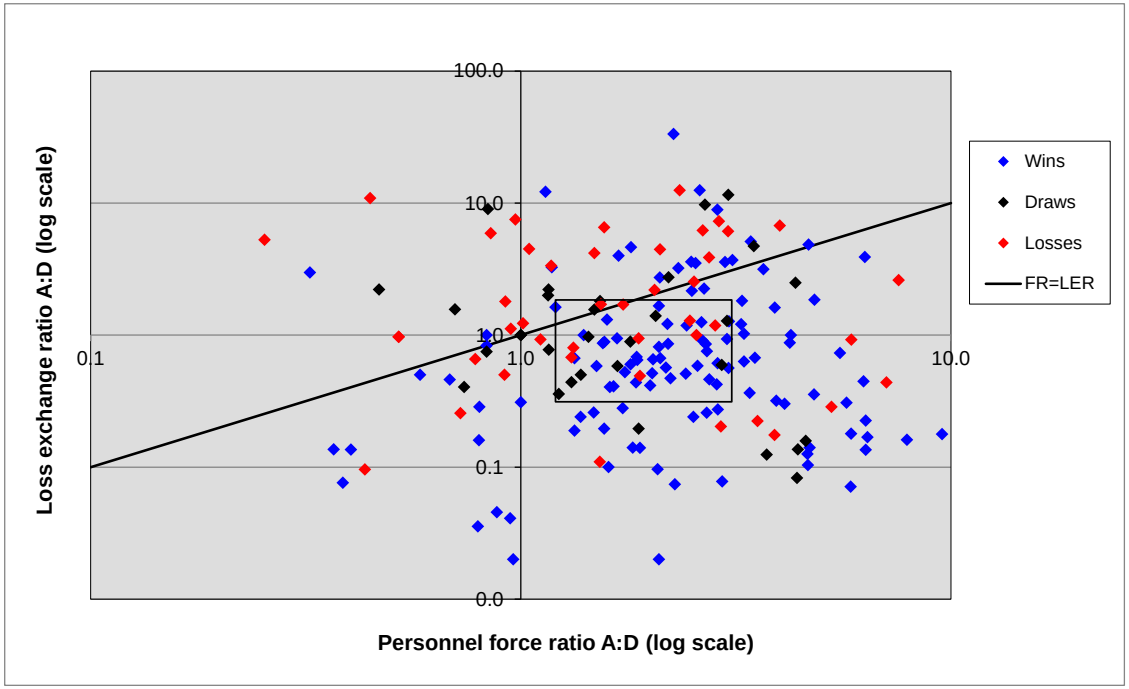
- Dupuy built the Helmbold database, 1980s
  - 660 battles, 1600–1982
  - displayed great variability
- Goodman and Young, CORDA, UK, 1990s
  - 218 battles, 1938–1991
- Dstl attempted to combine these in 2016
  - discarded Helmbold data points pre-1937
  - found substantial data compatibility and definition issues
  - needed more post-1991 data points

A recent Dstl study

Iraqi T-55 destroyed on Operation Desert Storm in 1991 (The Atlantic)

Developing a force planning tool

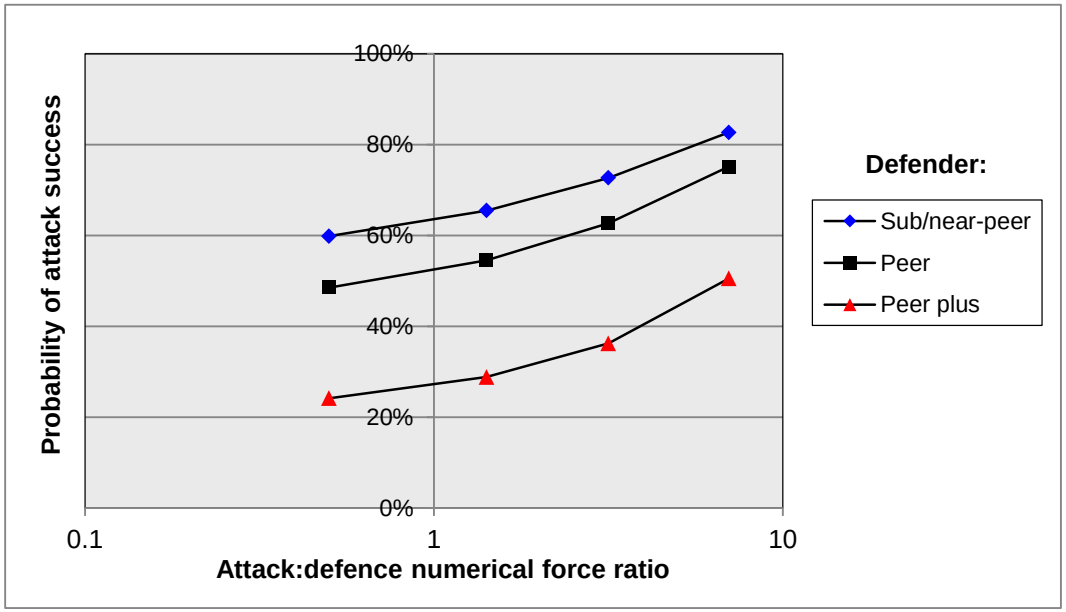
- A ‘Kirkpatrick plot’ for post-1937 Helmbold battles



The box bounds the 25%- and 75%-iles for both FR and LER

Conclusions on study

- Increasing force ratio increased P(success)
  - but it also *increased* the attacker's losses
  - confirms Rowland
- Great variability
  - human factors?
- Inconsistencies in losses data
  - merged database unsuccessful



Implications and conclusions

ARVN soldiers in Vietnam, 1967 (Tim Page)

Implications for combat models

- If modern battle is (largely) Lanchester linear …
  - or at least, closer to his linear than his square law …
- *Why persist with the square law?*
- Practical difficulties with the linear law
  - introduces another dimension, being target density
    - thus more data needed
  - models need to be re-written … cost and time
- Little understanding on which to build new models …
  - needing a bigger, cleaner, more modern HA database

Conclusions

- Weight of evidence favours Lanchester's linear model for dismounted ground combat, i.e. *most* combat
  - tank combat (probably) follows the square law
- Reasons and mechanisms still not totally clear
  - evidence points to most infantry fire being 'area fire'
- Combat modelling has not (in general) heeded this
  - due to inertia, or lack of awareness?
- Overall understanding of combat is not good

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Questions?

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