Australian Government
Department of Defence
Science and Technology

Military Simulation Analytics: Towards a Sensitivity Analysis when Conducting Analysis of Alternatives

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Improving Force Design Force Structure Review

Force Structure Review (FSR) process:

- Senior leadership applying military judgement over force options through seminar wargaming

Complexity of modern ops → difficult to rely on intuition for Force Design

- Many factors affect modern ops
- Difficulty in assessing impact of new capability (yet to be developed)
- Future wars fought differently to past

SR2 will deliver a sim capability for exploring & developing complex whole-of-force operating concepts

Operating Concepts for Exploration

- Force Level Electronic Warfare
- Maritime Force Defence
- Space Concepts
- Cooperative Engagement
- Information Age Combat Model
- Cyber Warfare
- New Operating Concepts

Involved Methods & Fields of Study

Future Operating Concepts

1. FLEW: Force Level EW
2. CEC: Cooperative Engagement Capability
3. Space Concepts
4. Maritime Force Defence
5. Cyber Warfare
6. New Concept 2

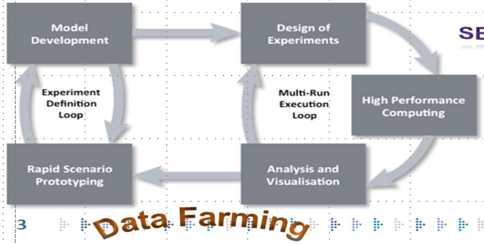
M&S

- Develop novel modelling strategies to represent abstract concepts in HPC-sim
- Resolving computational intractability in large scale simulation (many factors)

HPC | DoE

Analysis & Visualisation

- Develop new analysis strategies for high dim problem spaces (big data)
  - Many response vars.
  - Many design points
  - Many iterations @ a design point



3 DST Science and Technology for Safegua

Sensitivity Analysis and Analysis of Alternatives

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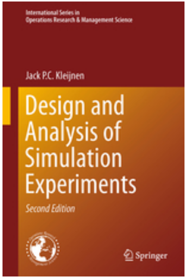
Sensitivity Analysis:

- Global sensitivity - Stochastic Kriging or Gaussian Process (optimisation)
- **Local sensitivity - low-order polynomials (main effects/two-way interactions)**
- Sub-system marginal contribution to operational effectiveness
- Combat multipliers (combined arms combat)
- Robustness of point-scenario insights to uncertain parameters



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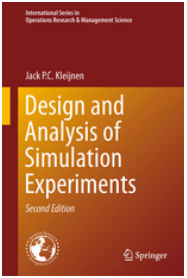
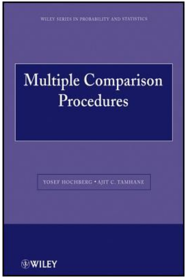
Analysis of Alternatives:

- Discrete set of competing systems (e.g. tender evaluation)
- All pairwise comparisons - generally produces only partial order
- Selection of the best or subset containing the best
- **Ranking analysis - score-based** or partition-based





Linear Regression

With m_i replications at each design point $\mathbf{x}_i$ fit using **OLS criterion**. The sum of squared residuals is:

$$SSR = \sum_{i=1}^{n}\sum_{r=1}^{m_i}(\hat{y}_i - w_{ir})^2 \text{ where } \hat{y}_i = \sum_{j=1}^{q} x_{ij}\hat{\beta}_j,\ i = 1, \ldots, n$$

$$= \sum_{i=1}^{n}\sum_{r=1}^{m_i}\left[(x_{ik}\hat{\beta}_k)^2 + 2x_{ik}\hat{\beta}_k \sum_{j=1;\neq k}^{q} x_{ij}\hat{\beta}_j + \left(\sum_{j=1;\neq k}^{q} x_{ij}\hat{\beta}_j\right)^2 - 2w_{ir}\sum_{j=1}^{q} x_{ij}\hat{\beta}_j + (w_{ir})^2\right].$$

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\end{aligned}
$$

Differentiating the SSR with respect to the $k-$th regression parameter gives:

$$
\frac{\partial SSR}{\partial \hat{\beta}_k} = 2\sum_{i=1}^{n} x_{ik} m_i \sum_{j=1}^{q} x_{ij}\hat{\beta}_j - 2\sum_{i=1}^{n} x_{ik} m_i \overline{w}_i, \quad k = 1,\ldots,q \quad \text{where} \quad \overline{w}_i = \sum_{r=1}^{m_i} w_{ir}/m_i.
$$

Normal equations for the OLS estimator: $X'MX\hat{\beta}^{OLS} = X'M\overline{\mathbf{w}}$.

Parameter Confidence Intervals

Since $\hat{\beta}_j^{OLS} = \sum_{i=1}^{n} L_{ji}\overline{w}_i$ where $L = (X'MX)^{-1}X'M$ and treating $\overline{w}_i$ as **random variables**:

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This **generalises and simplifies Kleijnen (2015)** who treated $m_i = m$ and $m_i \neq m$ separately.

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- $\sigma^2(W) \approx s^2(w) = \sum_{i=1}^{n}(m_i - 1)s^2(w_i)/(N - n)$ **pooling** n sample variance estimators, or
- $\sigma^2(W) \approx MSR = \sum_{i=1}^{n}\sum_{r=1}^{m_i}(\hat{y}_i - w_{ir})^2/(N - q)$ based on the **residuals** of the OLS regression.

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However Kleijnen (2015) used:

$$\text{``}MSR\text{''} = \sum_{i=1}^{n} m_i(\hat{y}_i - \overline{w}_i)^2/(N - q) = \sum_{i=1}^{n} m_i \left[\sum_{r=1}^{m_i}(\hat{y}_i - w_{ir})/m_i\right]^2 /(N - q).$$

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Kleijnen's (2015) lack-of-fit F-statistics are incorrect. The **correct, general, expression** is:

$$F_{N-q,N-n} = \frac{MSR}{s^2(w)} = \frac{\sum_{i=1}^{n}\sum_{r=1}^{m_i}(w_{ir} - \hat{y}_i)^2/(N-q)}{\sum_{i=1}^{n}\sum_{r=1}^{m_i}(w_{ir} - \overline{w}_i)^2/(N-n)}.$$

Common Random Numbers

With CRN as a **Variance Reduction Technique** $m_i = m$ and Σ_w approximated by **sample covariance** so:

$$var(\hat{\beta}_j^{OLS}) \approx \sum_{i=1}^{n} \sum_{i'=1}^{n} L_{ji} L_{ji'} \sum_{r=1}^{m} (w_{ir} - \overline{w}_i)(w_{i'r} - \overline{w}_{i'}) / [m(m-1)].$$

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Kleijnen (2015) proposed **alternative** method inspired by classical text **Law (2007)**.

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The two methods are **identical, not alternatives**.

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So the r-th jackknifed pseuduovalue $J_{j;r}$ **is identical to OLS estimator** based on r-th replication $\mathbf{w}_r$.

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- then **Law (2007) approach is in fact identical to both OLS and Jack-Knifing in Kleijnen (2015)**.
- **Confidence intervals** for the regression coefficients can be calculated from:

$$\hat{\beta}_j = \sum_{i=1}^{n} L_{ji}\overline{w}_i, \quad j = 1, \ldots, q$$

$$var(\hat{\beta}_j) \approx \sum_{i=1}^{n}\sum_{i'=1}^{n} L_{ji}L_{ji'} \sum_{r=1}^{m}(w_{ir} - \overline{w}_i)(w_{i'r} - \overline{w}_{i'})/[m(m-1)], \quad j = 1, \ldots, q$$

$$L = (X'X)^{-1}X' \quad \text{and} \quad \overline{w}_i = \sum_{r=1}^{m} w_{ir}/m, \quad i = 1, \ldots, n.$$

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- Generate sample data $z_i = 1$ if reject $H_0 : \mu_{1i} = \mu_{0i}$ in favour of $H_1 : \mu_{1i} > \mu_{0i}$.
- $odds(\mathbf{x}) = P(Z = 1|\mathbf{x})/[1 - P(Z = 1|\mathbf{x})]$ and $\exp(\hat{\beta}_j) = odds(\mathbf{x} + \mathbf{e}_j)/odds(\mathbf{x})$.

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Simple counterexample:

- If $P(Z=1|\mathbf{x}) = 1/3$ and $\hat{\beta}_j = 1 \rightarrow odds(\mathbf{x}) = 1:2$ and $odds(\mathbf{x}+\mathbf{e}_j) = 2.72:2 = 1.36:1$.
- Preference (decision) **has changed** from alternative 0 to alternative 1.
- If, however, $P(Z=1|\mathbf{x}) = 2/3$ and $\hat{\beta}_j = 1$. Then $odds(\mathbf{x}) = 2:1$ and $odds(\mathbf{x}+\mathbf{e}_j) = 5.44:1$.
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Unlikely that logistic regression (or **multinomial regression** for the general $K > 2$ case) will work.

Ranking Sensitivity Measure

Score-based method of Villacorta & Sáez (2015):

$$s_{ki} = \sum_{j=k+1}^{K} z_{kji} \quad \text{where} \quad z_{kji} = \begin{cases} +1, & \text{if accept } H_1 : \mu_{ki} > \mu_{ji} \\ 0, & \text{if accept } H_0 : \mu_{ki} = \mu_{ji} \\ -1, & \text{if accept } H_1 : \mu_{ki} < \mu_{ji} \end{cases}$$

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How to measure **similarity/distance** between $\mathbf{s}_i$ and $\mathbf{s}_{i'}$?

- **Convert to ranks**: $\sigma_i(k) = rank(s_{ki}, \mathbf{s}_i)$ so $\sigma_i(\cdot)$ is a permutation of $\{1, \ldots, K\}$.
- **Weighted Spearman's Footrule**: $\delta^F_{ii'} = \sum_{k=1}^{K} d_{ii'k} p_{ii'k} |\sigma_i(k) - \sigma_{i'}(k)|$. (Dolgun *et al.*, 2018).
- **Inter-rater disagreement**: $d_{ii'k} = \left(\frac{|s_{ki} - s_{ki'}|}{2(K-1)} \right)^p$ (Gwet, 2014).
- **Head or Tail Position**: $p_{ii'k} = \left(\frac{\sigma_i(k) + \sigma_{i'}(k)}{2} \right)^{-1/K}$ (Kumar & Vassilvitskii, 2000).

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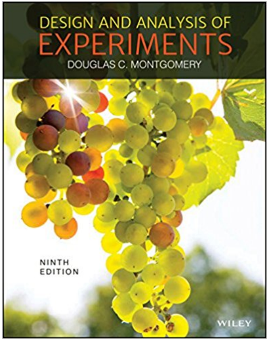
This provides a scalar measure of **sensitivity of ranking vector** between two design points, $\mathbf{x}_i$ and $\mathbf{x}_{i'}$.

- How do we use that to **isolate** the sensitivity of ranking vector to an **individual parameter** x_j?

Full Factorial Design Ranking Regression

	$\mathbf{x}_{.1}$	$\mathbf{x}_{.2}$	$\cdots$	$\mathbf{x}_{.j}$	$\cdots$	$\mathbf{x}_{.K}$	y
$\mathbf{x}_{1.}$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{i_j+.}$	x_{i_j1}	x_{i_j2}	$\cdots$	$+1$	$\cdots$	x_{i_jK}	y_{i_j+}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{i_j-.}$	x_{i_j1}	x_{i_j2}	$\cdots$	-1	$\cdots$	x_{i_jK}	y_{i_j-}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{2^K.}$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

Table: Full Factorial Design for Univariate Response



Full Factorial Design Ranking Regression

	$\mathbf{x}_{.1}$	$\mathbf{x}_{.2}$	$\cdots$	$\mathbf{x}_{.j}$	$\cdots$	$\mathbf{x}_{.K}$	y
$\mathbf{x}_{1.}$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{i_j+.}$	x_{i_j1}	x_{i_j2}	$\cdots$	$+1$	$\cdots$	x_{i_jK}	y_{i_j+}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{i_j-.}$	x_{i_j1}	x_{i_j2}	$\cdots$	-1	$\cdots$	x_{i_jK}	y_{i_j-}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
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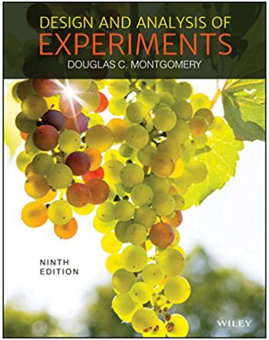

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Full Factorial Design Ranking Regression

	$\mathbf{x}_{.1}$	$\mathbf{x}_{.2}$	$\cdots$	$\mathbf{x}_{.j}$	$\cdots$	$\mathbf{x}_{.K}$	y
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$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{i_j+.}$	x_{i_j1}	x_{i_j2}	$\cdots$	$+1$	$\cdots$	x_{i_jK}	y_{i_j+}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{i_j-.}$	x_{i_j1}	x_{i_j2}	$\cdots$	-1	$\cdots$	x_{i_jK}	y_{i_j-}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
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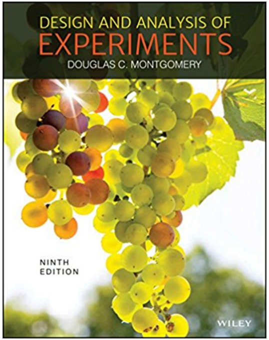

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$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{i_j+.}$	x_{i_j1}	x_{i_j2}	$\cdots$	$+1$	$\cdots$	x_{i_jK}	y_{i_j+}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{i_j-.}$	x_{i_j1}	x_{i_j2}	$\cdots$	-1	$\cdots$	x_{i_jK}	y_{i_j-}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{2^K.}$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

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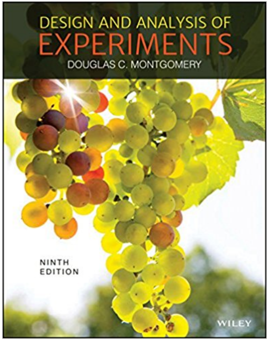

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	$\mathbf{x}_{.1}$	$\mathbf{x}_{.2}$	$\cdots$	$\mathbf{x}_{.j}$	$\cdots$	$\mathbf{x}_{.K}$	y
$\mathbf{x}_{1.}$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{i_j+.}$	x_{i_j1}	x_{i_j2}	$\cdots$	$+1$	$\cdots$	x_{i_jK}	y_{i_j+}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{i_j-.}$	x_{i_j1}	x_{i_j2}	$\cdots$	-1	$\cdots$	x_{i_jK}	y_{i_j-}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{2^K.}$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

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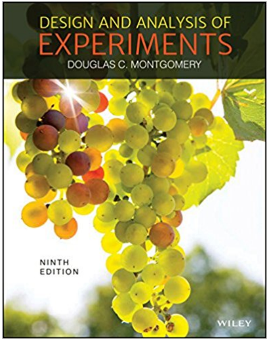

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- Key observations: Only involves **simulation output** δ; **other parameters *ceteris paribus***.
- For Full Factorial design **simply replace** $\delta_{i_j+;i_j-}$ with $\delta^F_{i_j+;i_j-} = \sum_{k=1}^K w_{i_j+;i_j-;k} |\sigma_{i_j+}(k) - \sigma_{i_j-}(k)|$.

Fractional Factorial Design Ranking Regression?

	$\mathbf{x}_{.1}$	$\mathbf{x}_{.2}$	$\cdots$	$\mathbf{x}_{.j}$	$\cdots$	$\mathbf{x}_{.K-p}$	$\mathbf{x}_{.K-p+1}$	$\cdots$	$\mathbf{x}_{.K}$	y
$\mathbf{x}_{1.}$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{i_j+.}$	x_{i_j1}	x_{i_j2}	$\cdots$	$+1$	$\cdots$	x_{i_jK-p}	$x_{i_j+K-p+1}$	$\cdots$	x_{i_j+K}	y_{i_j+}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{i_j-.}$	x_{i_j1}	x_{i_j2}	$\cdots$	-1	$\cdots$	x_{i_jK-p}	$x_{i_j-K-p+1}$	$\cdots$	x_{i_j-K}	y_{i_j-}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{2^{K-p}.}$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

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Fractional Factorial Design Ranking Regression?

	$\mathbf{x}_{.1}$	$\mathbf{x}_{.2}$	$\cdots$	$\mathbf{x}_{.j}$	$\cdots$	$\mathbf{x}_{.K-p}$	$\mathbf{x}_{.K-p+1}$	$\cdots$	$\mathbf{x}_{.K}$	y
$\mathbf{x}_{1.}$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{i_j+.}$	x_{i_j1}	x_{i_j2}	$\cdots$	$+1$	$\cdots$	x_{i_jK-p}	$x_{i_j+K-p+1}$	$\cdots$	x_{i_j+K}	y_{i_j+}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{i_j-.}$	x_{i_j1}	x_{i_j2}	$\cdots$	-1	$\cdots$	x_{i_jK-p}	$x_{i_j-K-p+1}$	$\cdots$	x_{i_j-K}	y_{i_j-}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{2^{K-p}.}$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

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	$\mathbf{x}_{.1}$	$\mathbf{x}_{.2}$	$\cdots$	$\mathbf{x}_{.j}$	$\cdots$	$\mathbf{x}_{.K-p}$	$\mathbf{x}_{.K-p+1}$	$\cdots$	$\mathbf{x}_{.K}$	y
$\mathbf{x}_{1.}$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{i_j+.}$	x_{i_j1}	x_{i_j2}	$\cdots$	$+1$	$\cdots$	x_{i_jK-p}	$x_{i_j+K-p+1}$	$\cdots$	x_{i_j+K}	y_{i_j+}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{i_j-.}$	x_{i_j1}	x_{i_j2}	$\cdots$	-1	$\cdots$	x_{i_jK-p}	$x_{i_j-K-p+1}$	$\cdots$	x_{i_j-K}	y_{i_j-}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{2^{K-p}.}$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

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Fractional Factorial Design Ranking Regression?

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$\mathbf{x}_{1.}$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{i_j+.}$	x_{i_j1}	x_{i_j2}	$\cdots$	$+1$	$\cdots$	x_{i_jK-p}	$x_{i_j+K-p+1}$	$\cdots$	x_{i_j+K}	y_{i_j+}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{i_j-.}$	x_{i_j1}	x_{i_j2}	$\cdots$	-1	$\cdots$	x_{i_jK-p}	$x_{i_j-K-p+1}$	$\cdots$	x_{i_j-K}	y_{i_j-}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{2^{K-p}.}$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

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$\mathbf{x}_{1.}$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{i_j+.}$	x_{i_j1}	x_{i_j2}	$\cdots$	$+1$	$\cdots$	x_{i_jK-p}	$x_{i_j+K-p+1}$	$\cdots$	x_{i_j+K}	y_{i_j+}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{i_j-.}$	x_{i_j1}	x_{i_j2}	$\cdots$	-1	$\cdots$	x_{i_jK-p}	$x_{i_j-K-p+1}$	$\cdots$	x_{i_j-K}	y_{i_j-}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\mathbf{x}_{2^{K-p}.}$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

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- For first $K - p$ parameters, **simply ignore** remaining p columns and i_j+, i_j- chosen as before.
- For $K - p + j$-th parameter, use its column for one of the first $K - p$ columns whose parameter was used to construct it (via the **generator words**).

Summary

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Sensitivity Analysis via low-order polynomial meta-models fit using OLS regression and Factorial designs:

1. Combat simulations **often depart from (all) standard NIID assumptions**:
   - Non-independently distributed (via use of CRN).
   - Non-identically distributed (variance depends on design point).
   - Non-normally distributed (skewness).
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Sensitivity Analysis of Analysis of Alternatives **new research topic**:

1. Borrow distance **metrics from Information Retrieval** algorithm comparisons.
2. Exploit **δ-structure and *ceteris paribus* principle** of Full Factorial designs.
3. **Future work**: test effectiveness of approach; examine other orthogonal/balanced designs; improve ranking sensitivity measure.